

# COMPONENTS OF INTEGRATION THEORY

Every notion of integration consists of the following five parts.

I. THE DEFINITION. The integral is defined as a limit of a sequence of sums. These sums are motivated by the desire to approximate some physical quantity that can be easily computed in simple situations. For example, the formula for the area of a rectangle is simple. To define the area of the region under the graph of a function defined on an interval  $[a, b]$ , this area was approximated by a large number of (very narrow) rectangles, the areas of these rectangles was added together and then the limit was taken as the bases of these rectangles tended to 0.

II. A LARGE CLASS OF INTEGRABLE FUNCTIONS. Continuing with the above example of every function continuous on  $[a, b]$  is integrable as is every function increasing on  $[a, b]$  and every function decreasing on  $[a, b]$ .

III. PROPERTIES OF THE INTEGRAL. Among the list of properties is always that the sum of two integrable functions is integrable and the integral of the sum is the sum of the integrals. Also a constant times an integrable function is integrable and the integral of the constant times the function is the constant times the integral of the function.

IV. A COMPUTATIONAL THEOREM. In the case of the integral of a function defined on  $[a, b]$  this theorem is the Fundamental Theorem of Calculus.

V. OTHER APPLICATIONS OF THE INTEGRAL. As was seen in earlier work the integral can be used to compute volumes, average values, work and several other quantities.