Review List

1) Show that the following are correct:

$$\lim_{n\to\infty} \frac{3}{\sqrt{2n+1}} = 0$$

$$\lim_{n\to\infty} \frac{2n+5}{3n+2} = \frac{2}{3}$$

2) Explain why $\sqrt{5}$ is not a rational number. Explain why $\sqrt{2}+\sqrt{5}$ is not a rational number (hint: let $q = \sqrt{2}+\sqrt{5}$ and consider $(q-\sqrt{2})^2 = 5$. Show that q rational implies $\sqrt{2}$ is rational).

3) If $\sum_{i=1}^{\infty} a_i$ converges and $\sum_{i=1}^{\infty} b_i$ diverges, does $\sum_{i=1}^{\infty} \left(2\,a_i - b_i\right)$ converge or diverge?

4) Is the set

$$\left\{ \frac{m}{n+m} \middle| n, m \in \mathbb{N} \right\}$$

bounded above? closed? open? What are its limit points? Briefly explain why $[2,3)$ is neither open nor closed.

5) Is $a_n = 1 + \frac{(-1)^n}{\sqrt{n}}$ a Cauchy sequence? (hint: you don't need to use ϵ inequalities!)

6) Use the monotone convergence theorem to explain why

$$x_1 = 1, \qquad x_n = (4\,x_{n-1})^{\frac{1}{3}}$$

is a convergent sequence. What is its limit?

7) If $\sum_{i=1}^{\infty} a_i$ converges, explain why $\sum_{i=1}^{\infty} a_i^n$ converges for any $n \in \mathbb{N}$ (this is similar to a homework problem, and done the same way).

8) Let a_n be a monotone increasing sequence. When is $\{a_n\}$ a closed subset of $\mathbb{R}$?

9) Let $A \subset B \subset \mathbb{R}$. Show that $\overline{A} \subset \overline{B}$.

10) Call $S \subset \mathbb{R}$ complete if every $(s_i) \subset S$ that is a Cauchy sequence in $\mathbb{R}$, has its limit in S. What subsets of $\mathbb{R}$ are complete?