Review for Math 152H-1 Test #2

Review Problems: Please note: these are meant to be challenging!

1. Calculate the follwing limits:

$$\lim_{x\to 3} \frac{4\sqrt{1+x}-5-x}{(x-3)^2} \qquad\qquad \lim_{x\to \frac{\pi}{2}^+} (x-\frac{\pi}{2})\, tan\, x$$

2. Calculate the following anti-derivatives:

$$\int 4\,x^{\frac{3}{2}}+5-\frac{2}{x^3}\,dx \qquad\qquad \int \frac{1}{(3+x)^2}+x^2\,sin^2(x^3)\,dx$$

$$\int \frac{sec^2\big((3x+4)^{\frac{3}{4}}\big)}{(3x+4)^{\frac{1}{4}}}\,dx \qquad\qquad \int \frac{x^2-1}{x^{\frac{3}{2}}-\sqrt{x}}\,dx$$

$$\int x^3\sqrt{1+x}\,dx$$

3. Find y' at a point (x, y) satisfying $x^2 - 3x\sqrt{1+y} + y^2 = 1$.

4. Find y' when $x^2 + 3xy - y^2 = 3$. Where are the horizontal tangents to the curve? Where are the vertical tangents? Are there any points where there is no unique tangent line? (Check if these are on the curve!!)

5. Find the linearization and quadratic approximation to $f(x) = \sqrt{x} - x$ at $x = 4$. Is the graph increasing or decreasing near this point? Is it concave up or concave down?

6. Find the linearization and quadratic approximation to a function satisfying $f'(x) = \frac{1}{1+f(x)}$, $f(3) = -2$ at $x = 3$. If you could graph $f(x)$ would the function be increasing or decreasing at $x = 3$?

7. Where is $g(x) = \frac{x}{1-x^2}$ increasing/decreasing? Where is it concave up/concave down? What are its inflection points, if any? Graph this function as completely as possible.

8. Draw a graph of the solution of

$$\frac{d}{dx}\,y = \sqrt{x}(4-2x^{\frac{3}{2}}),\ \ y(1) = \sqrt{2}$$

   Find the solution.

9. Find and classify as local max/local min all the critical points of $f(x) = x^4 - 6x^3 + 5$. Where are the absolute max/min for $f(x)$ on the interval $[-1, 5]$.

10. A farmer wants to enclose a plot of land by a straight river. This plot should have one side on the river and be in the shape of a rectangle. He will then subdivide the plot into two equal pieces, each touching the river. Due to a slope it costs twice as much to build a fence perpendicular to the river then it does to build parallel to the river. He wants to have at least 294 ft^2 of fenced pasture when he is done. What are the dimensions of the plot that will minimize his cost? He wishes to put sheep in one part of the pasture, and they need to be able to graze up to 10 ft away from the river. What dimensions will minimize his cost subject to this additional restriction?

11. What is the smallest volume of a right circular cone which encloses a sphere of radius 2?

12. Hot water (100 degrees) flows into a tub at a rate of $0.4\,gal/min$. In the tub already are 10 gallons of 40 degree water. Assume that the temperature of the resulting mix is given by $\frac{1}{V}\big(100 \cdot v_h + 40 \cdot v_c\big)$ where $V = v_c + v_h$ and $v_c = 10$ is the volume of cold water mixed with v_h gallons of hot water. How fast is the temperature changing 5 minutes after we begin adding the hot water?

13. You are standing $100 ft$ from a building. A ball is dropped from the top of the building and falls along the side of the building. At time t it will have fallen $5\,t^2$ feet. You watch the ball fall and record that the angle you see it at is decreasing at a rate of $\frac{1}{\sqrt{2}}$ rad/s when the angle is $\frac{\pi}{4}$. How tall is the building? (Assume that your height is negligible).

14. Explain why $x = \frac{1}{1+\sqrt{x}}$ has one solution of the interval $[0, \infty)$.

15. Use the Mean Value Theorem to explain why a polynomial $P(x) = a_0\,x^3 + a_1\,x^2 + a_2\,x^1 + a_3$ has at most 3 roots. Use this to explain why a fourth degree polynomial $a_0x^4 + a_1x^3 + \ldots + a_4$ has at most four roots. If you keep doing this, how do you show that $a_0\,x^n + a_1\,x^{n-1} + \ldots a_{n-1}\,x + a_n$ has at most n roots?