

Any of the following exercises are fair game for your final oral exam. I suggest that you write up your solutions neatly in your own handwriting to consult during that exam! All assigned book exercises are from the textbook available here:

<https://users.math.msu.edu/users/iwenmark/Teaching/MTH994/Fall2022/HDP-book.pdf>.

#### PROBLEMS ASSIGNED FROM CHAPTER 1

1. Write down the density for the result of a fair 6-sided die roll using Diracs.
2. Show that if  $X$  and  $Y$  are independent random numbers, then  $\mathbb{E}[XY] = \mathbb{E}[X]\mathbb{E}[Y]$ .
3. Let  $\{X_j\}_{j \in \mathbb{N}} \subset \mathbb{R}$  be a sequence of independent random numbers with  $\mathbb{E}[X_j] = \mu$  and bounded variance  $\text{Var}[X_j] \leq \sigma^2$  for all  $j \in \mathbb{N}$ . Use the Chebyshev Inequality to argue that

$$\mathbb{P} \left[ \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{j=1}^n X_j = \mu \right] = 1.$$

4. Do exercises 1.2.2 and 1.2.3 from the book.

#### PROBLEMS ASSIGNED FROM CHAPTER 2

5. Do exercise 2.3.5 from the book. Some additional hints were given in class (check your notes).
6. Do exercise 2.2.10 from the book.
7. Check that mean 0 Gaussian random variables are sub-gaussian, and compute their sub-gaussian norm in terms of their variance. Also verify that bounded random variables are sub-gaussian.
8. Do exercise 2.5.9 from the book.
9. Do exercise 2.6.5 from the book.
10. Digest the proof of Lemma 2.6.8 and be able to reproduce it.
11. Do exercises 2.7.10 and 2.7.11 from the book.
12. Prove Theorem 2.8.4 by doing exercises 2.8.5 and 2.8.6 from the book.