Last Time: Defined $\varepsilon(\vec{x},\vec{y}) \in [\gamma_a \cup \gamma_b] \in H_1(\Sigma) \big/ \mathrm{Span}[\vec{\alpha}] + \mathrm{Span}[\vec{\beta}]$

Where, γ_a is a g-tuple of arcs along the α-curves (one for each α_i) connecting $\vec{x}$ to $\vec{y}$

+ γ_b " " " " " β-curves " " β_i " $\vec{y}$ " $\vec{x}$.

Prop. $\varepsilon(\vec{x},\vec{y}) = 0 \iff \pi_2(\vec{x},\vec{y}) \neq \emptyset$

If $\varepsilon(\vec{x},\vec{y}) = 0$ then $\pi_2(\vec{x},\vec{y}) \leftrightarrow \pi_2(\vec{x},\vec{x}) \cong \mathbb{Z} \oplus H_2(Y^3)$

$$H_2(Y^3) \cong \ker\left(\mathrm{Span}[\vec{\alpha}] + \mathrm{Span}[\vec{\beta}] \xrightarrow{i_*} H_1(\Sigma)\right)$$

$H_2(Y) = 0 \Rightarrow \pi_2(\vec{x},\vec{y}) = \mathbb{Z}$ if non-empty

$H^1(Y) = 0 \iff H^1(Y;\mathbb{Q}) = 0$.

$\iff H_*(Y;\mathbb{Q}) \cong H_*(S^3;\mathbb{Q})$ Rational Homology Spheres

$\varepsilon(\vec{x},\vec{y}) \in H_1(\Sigma) \big/ \mathrm{Span}[\vec{\alpha}] + \mathrm{Span}[\vec{\beta}]$

$\cong$

$H_1(Y) \cong H^2(Y)$

This is because $Y = \Sigma \times I \cup (\alpha\text{-handles}) \cup (\beta\text{-handles})$

$H_1(Y) = H_1(\Sigma \times I) = H_1(\Sigma) \big/$ Relations generated by the α-curves + β-curves.

Note: The fact that the obstruction to having a Whitney disk connecting $\vec{x}$ to $\vec{y}$ lives in $H^2(Y)$

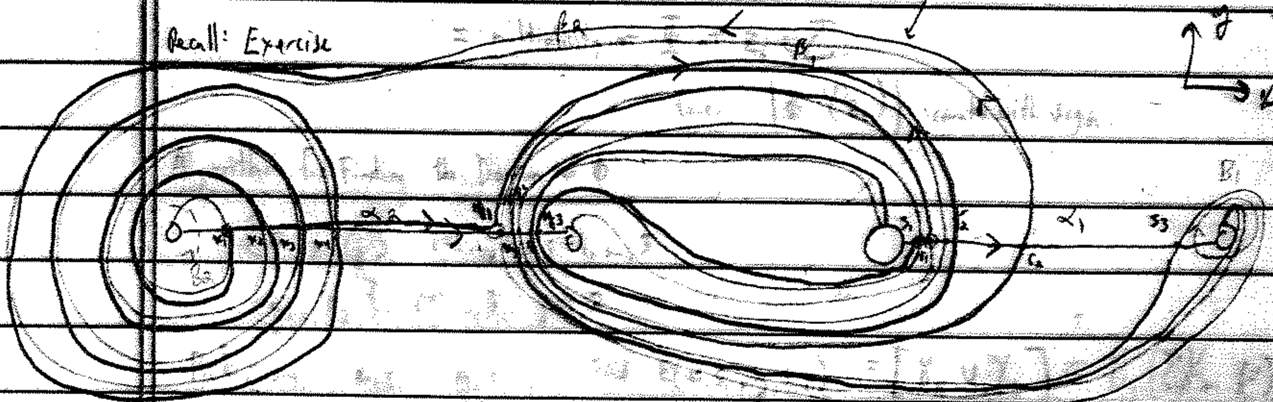
$$\Rightarrow CF^\circ(Y) = \bigoplus_{\eta \in H^2(Y)} CF^\circ(Y,\eta)$$

Recall: $H^2(Y) \xleftrightarrow{1:1} \mathrm{Spin}^c(Y)$

Recall: Exercise

$\gamma_\alpha \cup \gamma_\beta$

Orientation of Surface



Claim: $H_1(Y) = \mathbb{Z} = \mathbb{Z}/0\mathbb{Z} = H_1(S^3_0(K))$

Pf. $H_1(\Sigma) = \mathbb{Z}^4 = \mathbb{Z}\langle\alpha_1\rangle \oplus \mathbb{Z}\langle\alpha_2\rangle \oplus \mathbb{Z}\langle B_1\rangle \oplus \mathbb{Z}\langle B_2\rangle$

$\alpha_1 \cdot B_1 = 1$ $\quad \alpha_2 \cdot B_1 = 0$

$\alpha_1 \cdot B_2 = 0$ $\quad \alpha_2 \cdot B_2 = 1$

So we calculate, $\alpha_1 \cdot \alpha_1$

$\alpha_2 \cdot \alpha_2$

$\beta_1 \cdot \alpha_1 = 1 \qquad \beta_1 \cdot B_1 = 0$

$\beta_1 \cdot \alpha_2 = 1 \qquad \beta_1 \cdot B_2 = 0$

$\beta_2 \cdot \alpha_1 = -2 \qquad \beta_2 \cdot B_1 = -1$

$\beta_2 \cdot \alpha_2 = -2 \qquad \beta_2 \cdot B_2 = -1$

So $H_1(\Sigma) / \mathrm{Span}[\vec{\alpha}] + \mathrm{Span}[\vec{\beta}] = \mathbb{Z}\langle B_1 \rangle \oplus \mathbb{Z}\langle B_2 \rangle \big/ \langle [\beta_1] = 0, [\beta_2] = 0 \rangle$

$[\beta_1] = [-B_1 - B_2] = 0 \quad \Rightarrow \quad \mathbb{Z}\langle B_1 \rangle \oplus \mathbb{Z}\langle B_2 \rangle \big/ {-B_1} = B_2,\ 2B_1 = 2B_2 \cong \mathbb{Z}$

$[\beta_2] = [2B_1 + 2B_2 + \alpha_1 + \alpha_2] = 0$

<u>Def.</u> Given a Whitney disk $\phi \in \pi_2(\vec{x}, \vec{y})$, define the <u>domain of ϕ</u>, $D(\phi)$, to be

$$D(\phi) = \sum_i n_{z_i}(\phi) \cdot D_i \in \mathbb{Z}\left\langle \begin{matrix}\text{components of} \\ \Sigma \setminus \{\vec{\alpha} \cup \vec{\beta}\}\end{matrix} \right\rangle$$

i.e. Domains

Here, D_i is a domain, and $z_i \in D_i$.

In terms of pairs of maps, consider a

$$\left\{ \begin{matrix} F^2 \xrightarrow{\Phi} \Sigma \\ \downarrow p \\ \mathbb{D}^2 \end{matrix} \right\} \text{ corresponding to } \phi \in \pi_2(\vec{x}, \vec{y});$$

Then $$n_{z_i}(\phi) = \#_{\text{Alg.}} \left\{ \mathrm{Im}(\phi \times p) \in (\Sigma \times \mathbb{D}) \cap (z_i \times \mathbb{D}) \right\}$$

$= \text{multiplicity of } \Phi \text{ at } z_i \in \Sigma.$

(i.e. $|\Phi^{-1}(z_i)|$, counted with sign.

<u>Algorithm for Finding the Domain of ϕ</u>

$\{x_i, s_j\}$ $i = 1, \dots, 6$, $j = 1, \dots, 3$

$\{y_i, r_j\}$ $i = 1, \dots, 3$, $j = 1, 2$

Pick, say, $x_1 s_1$, $y_1 r_1$. Find $\varepsilon(x_1 s_1, y_1 r_1). = [\gamma_a \cup \gamma_b] \in H_1(\Sigma) / \mathrm{Span}[\vec{\alpha}] + \mathrm{Span}[\vec{\beta}]$

$\qquad\qquad \| $

$\mathbb{Z}\langle B_1 \rangle \oplus \mathbb{Z}\langle B_2 \rangle \big/ B_1 = -B_2$

$\gamma = \gamma_a \cup \gamma_b$.

$$\left.\begin{array}{ll} \gamma \cdot \alpha_1 = -1 & \gamma \cdot B_1 = 1 \\ \gamma \cdot \alpha_2 = -3 & \gamma \cdot B_2 = 0 \end{array}\right\} \Rightarrow [\gamma] = [B_1 + 3B_2 + \alpha_1]$$
$$= [B_1 + 3B_2]$$
$$= [2B_2] \neq 0.$$

So there's no Whitney disk connecting these two points.

Let's look at a pair of points we know will have a Whitney disk!

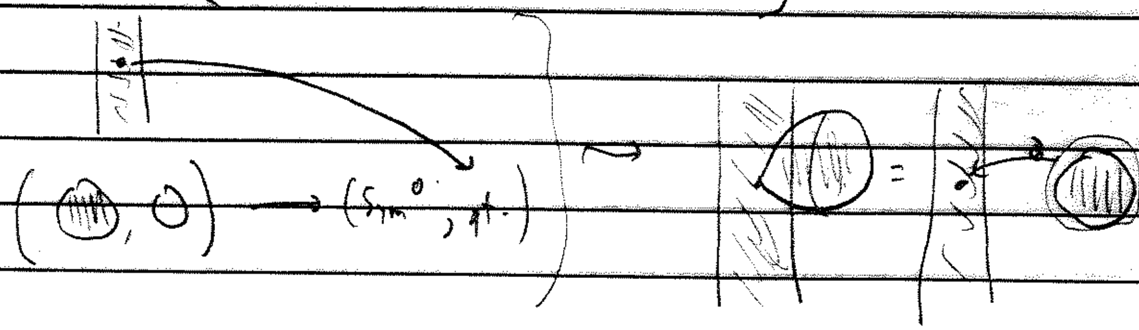
$$\pi_2(x_1 s_1, x_1 s_1) \neq \emptyset$$
$$\cong \mathbb{Z} \oplus H_2(Y) \cong \mathbb{Z} \oplus H^1(Y) \underset{\text{UCT}}{\cong} \mathbb{Z} \oplus \mathbb{Z} \cong \mathbb{Z}\langle[\Sigma]\rangle \oplus H_2(Y)$$

- First $\mathbb{Z}$-factor's worth of homotopy classes comes from $H_2(\Sigma)$
- Domain of constant $* \; n[\Sigma] \quad n \in \mathbb{Z}$

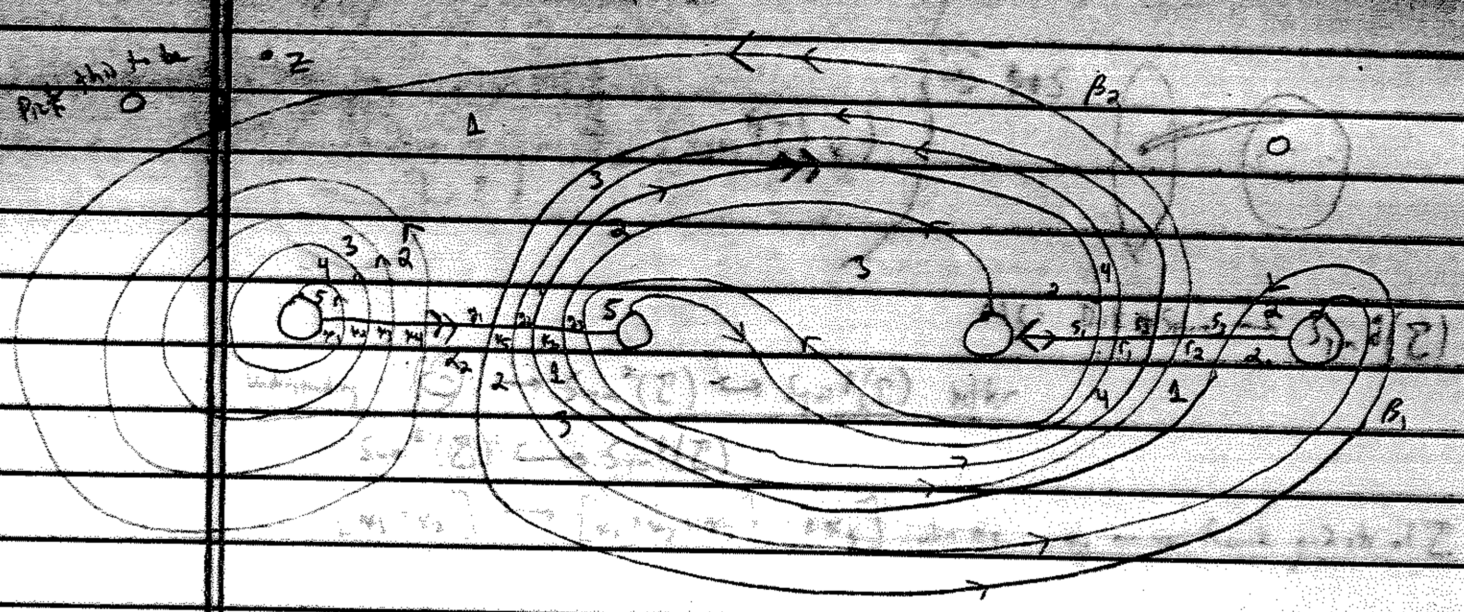
$$= n[\Sigma]$$
$$D(n[\Sigma])$$
$$\underbrace{\Sigma^2 \sqcup \Sigma^2 \sqcup \cdots \sqcup \Sigma^2}_{n} \sqcup \mathbb{D}^2 \xrightarrow{\Phi} \Sigma^2 \qquad + \mathrm{Id} \sqcup \cdots \sqcup \mathrm{Id}$$
$$\downarrow \qquad\qquad \text{pt.}$$
$$\mathbb{D}^2$$

$\underline{n=1}$: Want: $F^2 \longrightarrow \Sigma$, $\downarrow$ $\mathbb{D}^2$ $\qquad \Sigma \xrightarrow{\mathrm{Id.}} \Sigma$, Hyperelliptic involution $\downarrow$ 2:1 branched cover S^2 $\qquad S^2 \to \mathrm{Sym}^2(\Sigma) \subseteq \mathrm{Sym}^g(\Sigma)$, $p \longmapsto p \times z_1 \times \cdots \times z_{g-2}$

Now, suppose $\left\{\begin{array}{l} \mathbb{D}^2 \sqcup \cdots \sqcup \mathbb{D}^2 \longrightarrow \Sigma \quad (\mathbb{D}^2_i \to x_i) \\ \downarrow \mathrm{Id} \sqcup \mathrm{Id} \sqcup \cdots \sqcup \mathrm{Id} \\ \mathbb{D}^2 \end{array}\right\}$ represents the constant map inside $\pi_2(\vec{x}, \vec{x})$.



Matt [illegible] HFH 10/27/10



Last Time:
- Calculated $H_1(S^3_0(\text{trefoil})) \cong \mathbb{Z}$ from HD.
- Calculated $\mathcal{E}(0, \bullet)$ for a particular pair
- Defined the Domain $D(\phi)$ of a Whitney disk.
- In process of understanding $\pi_2(0,0) \cong \mathbb{Z} \oplus H_2(Y)$.

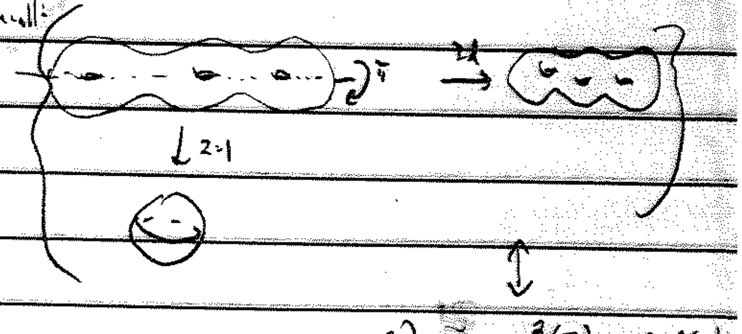
$\pi_2(0,0) \ni$ const.

$$0 = [x_1, \dots, x_g] \qquad \left\{ \begin{array}{c} \mathbb{D}_{x_1} \sqcup \cdots \sqcup \mathbb{D}_{x_g} \to \Sigma \\ \downarrow g:1 \\ \mathbb{D} \end{array} \right\} \leftrightarrow \text{constant}$$

$\pi_2(\mathrm{Sym}^2(\Sigma)) \cong \mathbb{Z}\langle S \rangle$

$$\begin{array}{c} \pi_2(0,0) \cong \mathbb{Z} \oplus H_2(Y) \\ \text{\rotatebox{90}{$\cong$}} \\ \mathbb{Z}\langle[\Sigma]\rangle \end{array}$$

Recall:



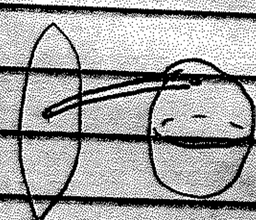
$$\underbrace{S^2 \to \mathrm{Sym}^2(\Sigma) \hookrightarrow \mathrm{Sym}^g(\Sigma)}_{S}$$

$$S \xrightarrow{\phi} (\mathrm{Sym}^g(\Sigma), *)$$

$\mathbb{Z}\langle[\Sigma]\rangle$

$$(\Sigma, p) \xrightarrow{S} (\mathrm{Sym}^g(\Sigma), *) \qquad 0 \in \mathbb{Z}\langle D_i \rangle$$

$\overset{\text{glue}}{\Longrightarrow} \phi * S$

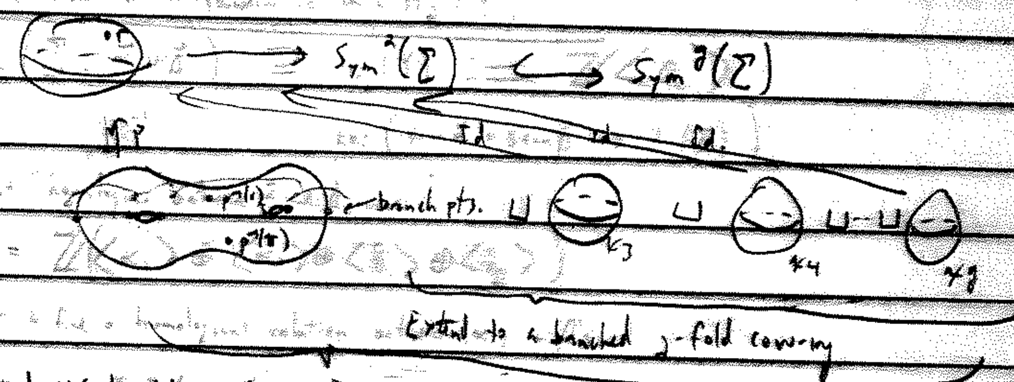


$\phi * S$: Disk # Sphere $\to \mathrm{Sym}^g(\Sigma)$

Understanding $\bigcirc \to \mathrm{Sym}^2(\Sigma) \hookrightarrow \mathrm{Sym}^g(\Sigma)$ better

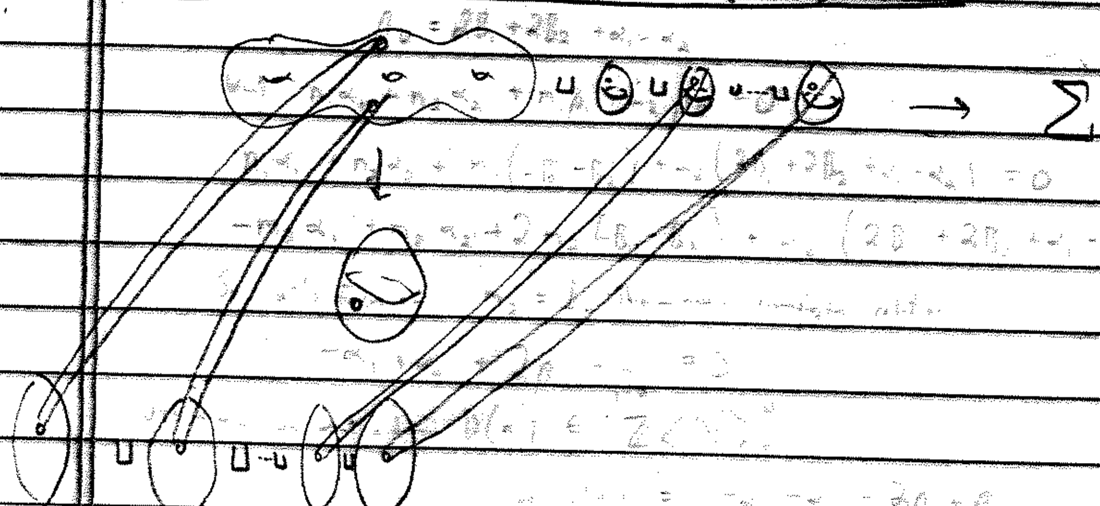
$$\mathrm{Sym}^2(\Sigma) \hookrightarrow \mathrm{Sym}^g(\Sigma)$$

$[x_1 \times x_2] \to [x_1 \times x_2 \times x_3 \times \cdots \times x_g]$, where $x_3, \dots, x_g$ are your favorite $g-2$ pts. of Σ.



Id. $\sqcup$ Const.$_{x_3}$ $\sqcup \cdots \sqcup$ Const.$_{x_g}$ $\to \Sigma$

So what is constant $* S$ in terms of $\{F \to \Sigma\}$?



$\mathbb{Z}\langle[\Sigma]\rangle$

$\mathbb{Z}\langle[\Sigma]\rangle$

$D(\text{constant}) = 0 \in \mathbb{Z}\langle D_i\rangle_{i=1}^{N}$

$D(\text{const} * S) = I = 1\cdot D_1 + 1\cdot D_2 + \cdots + 1\cdot D_N.$

$= 1\cdot[\Sigma]$

$H_2(S_0^3(K)) \cong \mathbb{Z} = \mathbb{Z}\langle[\hat{F}]\rangle$

F Seifert surface for K,
$\hat{F}$ the capped off surface after 0-surgery on K.

How do we see the domain corresponding to $\alpha \in H_2(Y)$?

$$H_2(\Sigma, \vec{\alpha}\cup\vec{\beta}) \supseteq H_2(Y) := \mathbb{Z}\langle a\rangle$$

$$\overset{\shortparallel}{\ker}\left(\operatorname{span}\vec{\alpha} + \operatorname{span}\vec{\beta} \to H_1(\Sigma)\right)$$

So we want "homologies between $\vec{\alpha}$ and $\vec{\beta}$ curves".

$$H_1(\Sigma) = \mathbb{Z}\left(\langle\alpha_1\rangle \oplus \langle\alpha_2\rangle \oplus \langle B_1\rangle \oplus \langle B_2\rangle\right)$$

Want to find a homological relation between α's + β's.

$\alpha_1 = \alpha_1$

$\alpha_2 = \alpha_2$

$\beta_1 = -B_1 - B_2$

$\beta_2 = 2B_1 + 2B_2 + \alpha_1 - \alpha_2$

Want: $n_1\alpha_1 + n_2\alpha_2 + m_1\beta_1 + m_2\beta_2 = 0$

$n_1\alpha_1 + n_2\alpha_2 + m_1(-B_1 - B_2) + m_2(2B_1 + 2B_2 + \alpha_1 - \alpha_2) = 0$

$-m_2\alpha_1 + m_2\alpha_2 + 2m_2(-B_1 - B_2) + m_2(2B_1 + 2B_2 + \alpha_1 - \alpha_2) = 0$ ✓

So let's focus on $m_2 = 1$. Then we have a homological relation:

$$-\alpha_1 + \alpha_2 + 2\beta_1 + \beta_2 = 0$$

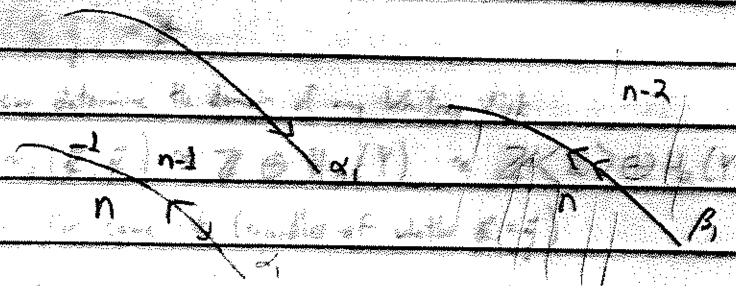
Ultimately, we want a domain $D(a) \in \mathbb{Z}\langle D_i\rangle_{i=1}^{N}$

$$\partial D(a) = -\alpha_1 + \alpha_2 + 2\beta_1 + \beta_2$$

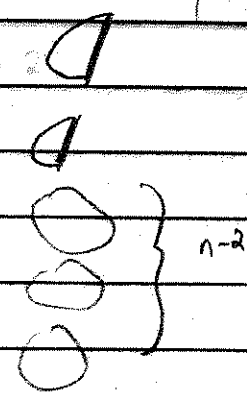
See Picture

$D(a)$ $\cong \mathbb{Z} \oplus H_2(Y) = \mathbb{Z}\langle\ \rangle$

$-\alpha \in \partial D(a) \Rightarrow$ $\qquad 2\beta_1 \in \partial D(a) \Rightarrow$



So in the end, we have a domain $D(a)$
whose boundary is $-\alpha_1 + \alpha_2 + 2\beta_1 + \beta_2$



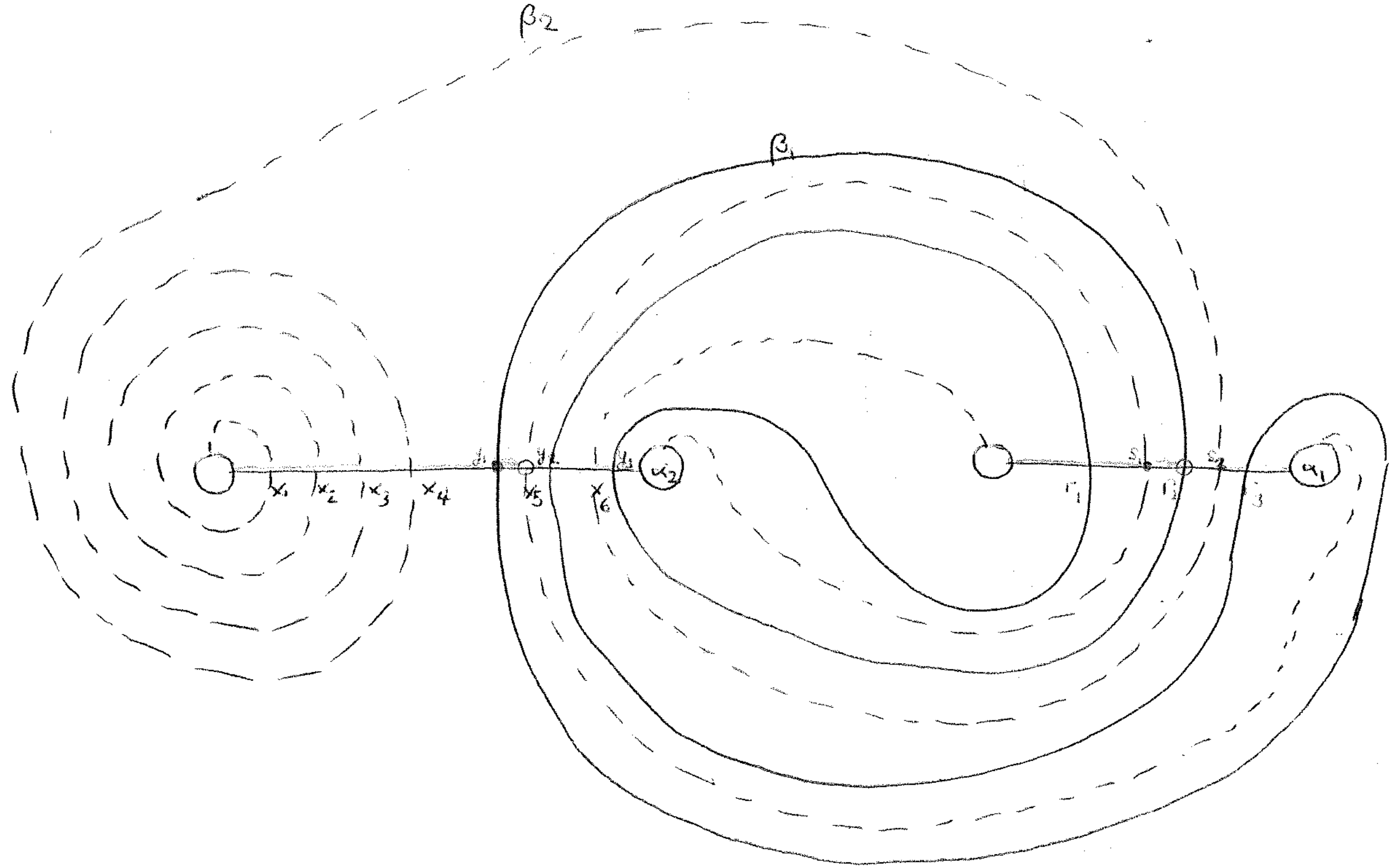
The only choice was multiplicity at z.

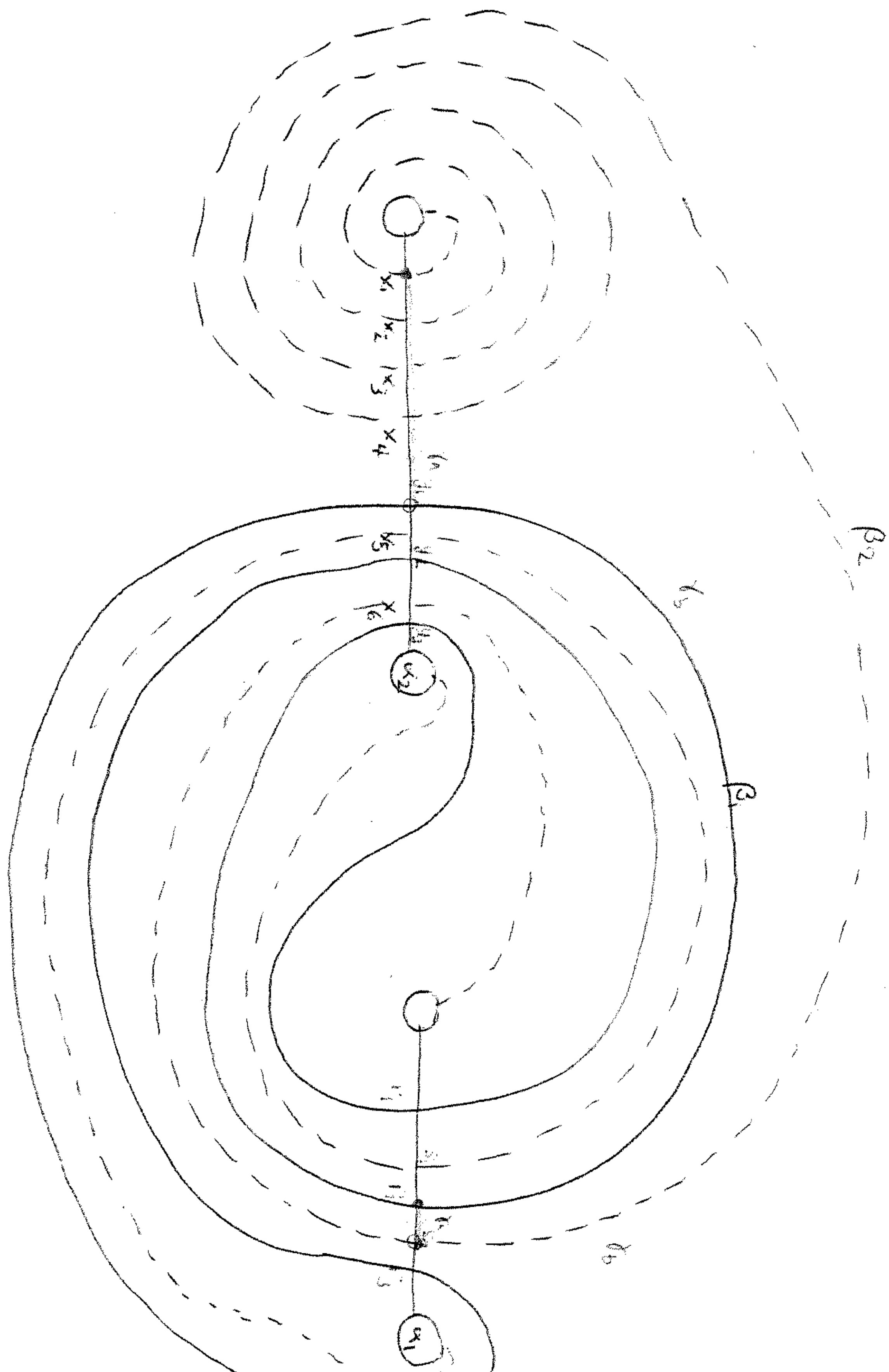
<u>Note:</u> $\pi_2(\circ, \circ) \cong \mathbb{Z} \oplus H_2(Y)$, but non-canonically.

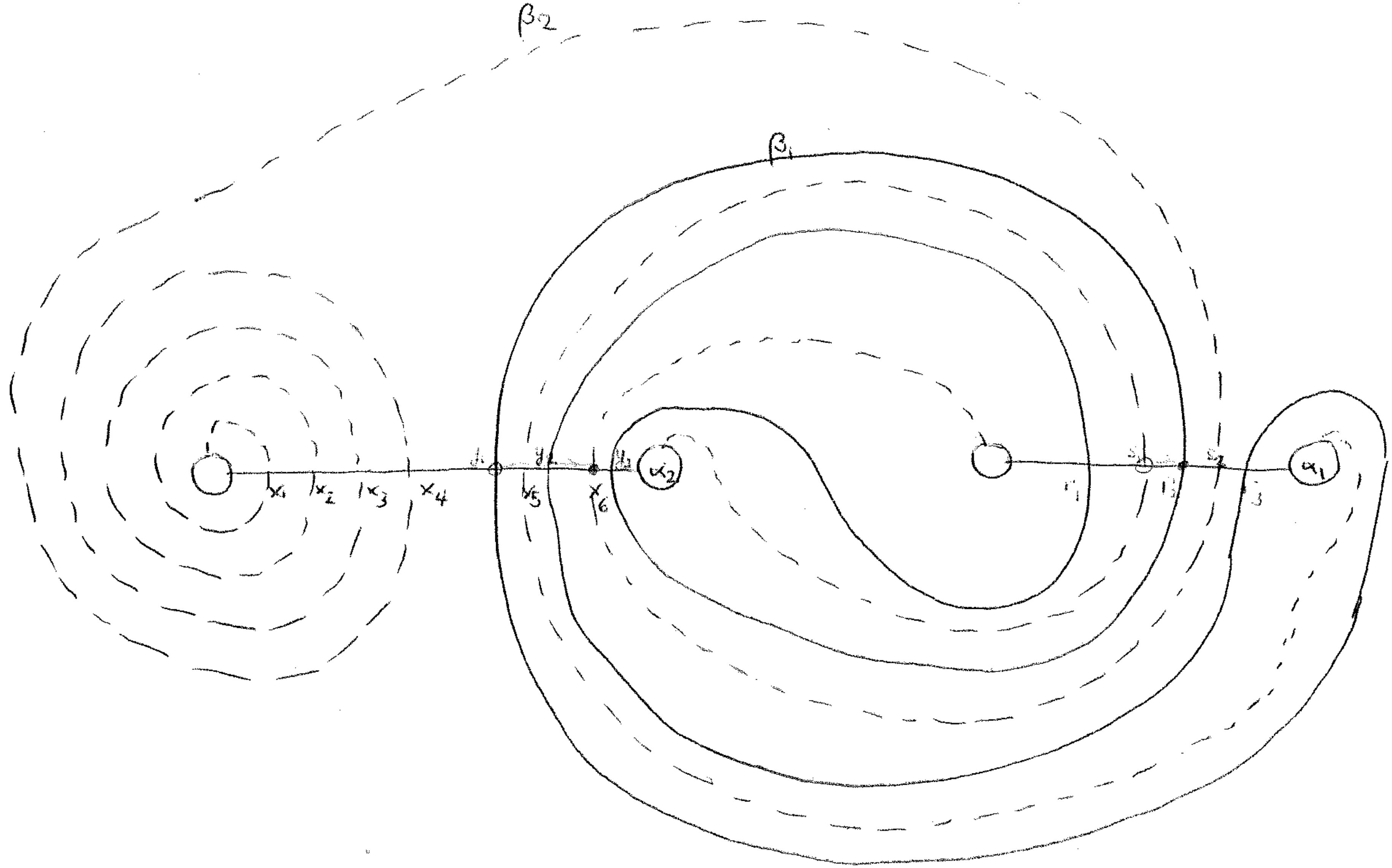
$$0 \to \mathbb{Z} \to \pi_2(\circ, \circ) \to H_2(Y) \to 0$$

Pick $n_z(\phi) = \text{mult}_z(D(\phi))$

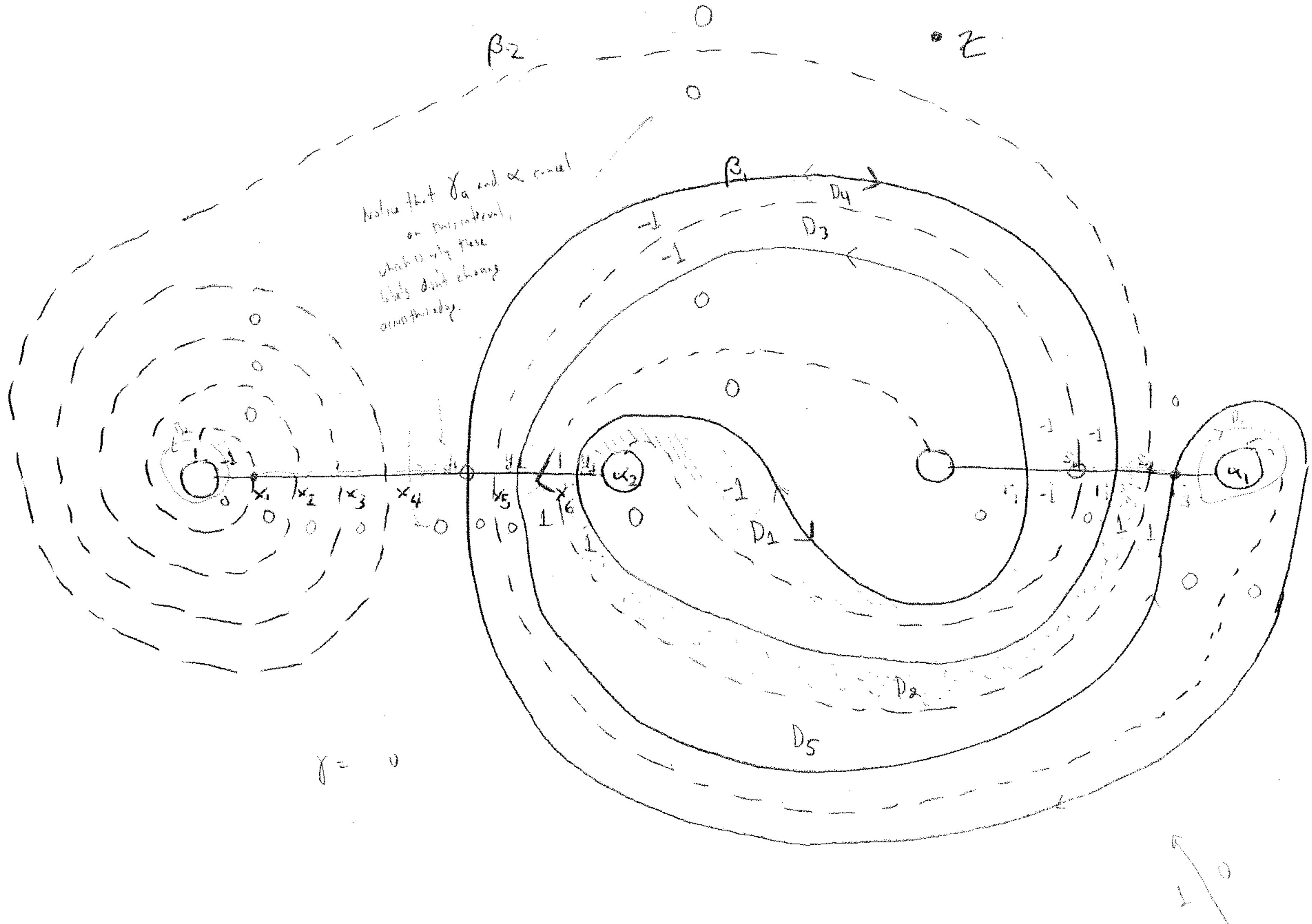
This picks out the isomorphism here


β_2
β_1
x_1
x_2
x_3
x_4
x_5
x_6
y_1
y_2
y_3
α_2
α_1
s_1
s_2
r_1
r_2
r_3


β_2
β_1
α_2
α_1
x_1
x_2
x_3
x_4
x_5
x_6



β_2
β_1
x_1
x_2
x_3
x_4
x_5
x_6
α_2
α_1


β_2
z
β_1
D_1
D_2
D_3
D_4
D_5
α_1
α_2
x_1
x_2
x_3
x_4
x_5
x_6
Notice that γ_a and α cancel on this interval, which is why these labels don't change across this edge.
$\gamma = 0$

HFH

Last Time: Given $\mathcal{H} = (\Sigma, \alpha, \beta)$, we know how to

① Enumerate generators for $CF^{\circ}(Y)$

(2) Determine if $\pi_2(\vec{x}, \vec{y}) = \emptyset$

(3) If $\pi_2(\vec{x}, \vec{y}) \neq \emptyset$, can determine the domain of any Whitney disk coming from splicing $\pi_2(\vec{x}, \vec{x}) \cong \mathbb{Z} \oplus H_2(Y) \cong \mathbb{Z}\langle\Sigma\rangle \oplus H_2(Y)$

Today:
- If $\pi_2(\vec{x}, \vec{y}) \neq \emptyset$, find $D(\phi)$ for some ϕ (regardless of whether $\vec{x} = \vec{y}$)
- In terms of $D(\phi)$, we'll calculate $\mu(\phi)$.
- Determine some sufficient conditions for $\# \widehat{\mathcal{M}}(\phi) = 1 \pmod 2$
- Spin^c-structures.

(See Handout, p. I)

$\bullet = y_1 s_1 \qquad \circ = x_5 r_2$

$\pi_2(\bullet, \circ) = ?$

$$\varepsilon(\bullet, \circ) = [\underbrace{\gamma_a \cup \gamma_b}_{= \gamma}] = [\quad] \in H_1(\Sigma) / \text{Span}[\alpha] + \text{Span}[\beta]$$
$$= 0$$

$\Rightarrow \pi_2(\bullet, \circ) \neq \emptyset$. What is a domain for some $\phi \in \pi_2(\bullet, \circ)$?

To find a domain, *in general*, we find a null-homology for

$$\gamma + \sum n_i \alpha_i + \sum m_i \beta_i = 0$$

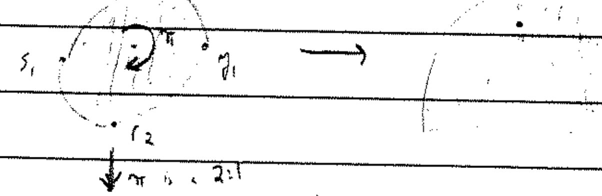
(which exists by the fact that $[\gamma] = 0 \in H_1(\Sigma) / \text{Span}[\alpha] + \text{Span}[\beta]$)

i.e. We want to find a cellular 2-chain

$$D(\phi) \in \mathbb{Z}\langle D_i \rangle \text{ s.t. } \partial D\phi = \gamma + \sum n_i \alpha_i + \sum m_i \beta_i.$$

For our specific choice, such a domain is provided by (put a 1 in this domain, 0 everywhere else)

To concretely see the Whitney disk in $\text{Sym}^2(\Sigma_2)$, consider the following pair of maps + surfaces:



(Branch pt. lifts to single pt. w/ multiplicity 2, which also corresponds to a pt. in $\text{Sym}^2(\Sigma)$.)

$\downarrow \pi$ is a 2:1 branched cover (over 1 pt.)

$$[\pm 4 B_2] \in \quad \text{But } H_1(\Sigma) = \frac{\mathbb{Z}\langle \alpha_1 \oplus \alpha_2 \oplus B_1 \oplus B_2 \rangle}{\langle \alpha_1 \rangle + \langle \alpha_2 \rangle + \langle 2B_1 = -2B_2 \rangle}$$

$$\neq$$
$$0.$$

Ex! $\varepsilon(x_1 r_2, y_1 s_1) = [\gamma] \in H_2(\Sigma)/\vec{\alpha}+\vec{\beta}$.

(See handout p. 3)

$$\gamma \cdot \alpha_2 = 1 \qquad \gamma \cdot B_2 = 0$$

$$\gamma \cdot \alpha_1 = 0 \qquad \gamma \cdot B_1 = 1$$

$$[\gamma] = [\alpha_1 - B_2] \in H_1(\Sigma)$$

$$[\gamma] = [-B_2] \neq 0 \text{ in } H_1(\Sigma)/\vec{\alpha}+\vec{\beta}.$$

Exercise: Determine $\varepsilon(\vec{x}, \vec{y})$ for all 24 $\vec{x} \in \mathbb{T}_\alpha \cap \mathbb{T}_\beta$.

Recall: $\varepsilon(\vec{x}, \vec{y}) + \varepsilon(\vec{y}, \vec{z}) = \varepsilon(\vec{x}, \vec{z})$

Maslov Index of $\phi \in \pi_2(\vec{x}, \vec{y})$ in terms of $D(\phi)$.

(Lipschitz - Section 4, Bourgois' Thesis)

$$D(\phi) \in \mathbb{Z}\langle D_i \rangle_{i=1}^{\text{Number of domains}}$$

To a domain, $D_i \in \Sigma \setminus \{\vec{\alpha} \cup \vec{\beta}\}$, we first associate its "Euler measure"

$$e(D_i) = \chi(D_i) - \frac{1}{4} \# \{\underbrace{\text{accute vertices on } \partial D_i}_{\text{Less than } \pi}\} - \frac{3}{4}\{\underbrace{\text{obtuse vertices on } \partial D_i}_{\text{Greater than } \pi.}\}$$

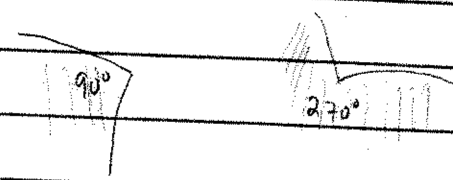
Matt Hedden HFH 10/14/10 Ron: Algorism - Extremely exciting, ...

Maslov Index Formula

Euler measure : e, $\hat{\chi}$ (piecewise)

Given a surface S with geodesic boundary, + corner pts. on ∂S, all of which are perpendicular intersections of geodesic arcs.

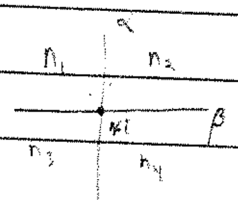
Def. $\hat{\chi}(S) := \chi(S) - \frac{1}{4}\#\begin{pmatrix}\text{accute } (90°) \\ \text{corners}\end{pmatrix} - \frac{3}{4}\#\begin{pmatrix}\text{obtuse } (270°) \\ \text{corners}\end{pmatrix}$.



For a domain $D_i \in \Sigma \setminus \{\vec{\alpha} \cup \vec{\beta}\}$, $\hat{\chi}(D_i) := \hat{\chi}(\overline{D_i}) = \chi(D_i) - \frac{1}{4}\#(\text{corner pts.})$

$$\hat{\chi}(D(\phi)) = \sum_{i=1}^{N} n_{p_i}(\phi) \cdot \hat{\chi}(D_i)$$

Point measure :



$n_{x_i}(D(\phi)) = \frac{1}{4}(n_1 + n_2 + n_3 + n_4)$

= Average of multiplicities of $D(\phi)$ in the 4 (not necessarily distinct) regions adjacent to x_i.

$$n_{\vec{x}}(D(\phi)) = \sum_{i=1}^{g} n_{x_i}(D(\phi))$$

Thm. (Lipschitz) For $\phi \in \pi_2(\vec{x}, \vec{y})$,

Maslov index $\mu(\phi) = \hat{\chi}(D(\phi)) + n_{\vec{x}}(D(\phi)) + n_{\vec{y}}(D(\phi))$. (*)

Exercise: Show that $\mu(\phi)$ defined by (*) satisfies

(1) Additivity: $\mu(\phi_1 * \phi_2) = \mu(\phi_1) + \mu(\phi_2)$

(2) "Invertibility" : $\mu(\phi^{-1}) = -\mu(\phi)$

(3) $\mu(\text{constant}) = 0$.

(4) $\mu(\phi * nS) = \mu(\phi) + 2n$

$S \in \pi_2(\text{Sym}^g)$, generator, $D(S) = [\Sigma]$

Ex: $\mu\left(y_1 \; x_5 \; s_1 \; r_2 \right) = \hat{\chi}(\cap) = \chi(\cap) - \frac{1}{4}\#(\text{corner pts.}) + n_\bullet(\cap) + n_\circ(\cap)$

$= 1 - \frac{1}{4}(4) + \left(n_{y_1}(\cap) + n_{s_1}(\cap)\right) + \left(n_{x_5}(\cap) + n_{r_2}(\cap)\right)$

$= 1 - 1 + \left(\frac{1}{4} + \frac{1}{4}\right) + \left(\frac{1}{4} + \frac{1}{4}\right)$

$= 1$

Ex: $\pi_2(x_1, r_3, y_1, s_1) \ni \phi$ (See p.4 of Handouts)

$\gamma \cdot \alpha_1 = 1 \qquad \gamma \cdot \beta_1 = 0$

$\gamma \cdot \alpha_2 = 1 \qquad \gamma \cdot \beta_2 = 1$

$1 \curvearrowleft 0 \qquad [\gamma] = [\beta_1 + \alpha_2] \in H_2(\Sigma)$

$\Rightarrow [\gamma - \beta_1 - \alpha_2] = 0 \in H_2(\Sigma)$

$\Rightarrow \exists$ Null-homology for $\gamma - \beta_1 - \alpha_2$

$\hat{\chi}(D(\phi)) = -1\,\hat{\chi}(D_1) + 1\,\hat{\chi}(D_2) + 1\,\hat{\chi}(D_5) - 1\,\hat{\chi}(D_3) - 1\,\hat{\chi}(D_4) = 0$

$n_{x_1, r_3}(D(\phi)) = \frac{1}{4}(\text{Sum of mult. around } x_1) + \frac{1}{4}(\text{Sum of mult. around } r_3)$

$= \frac{1}{4}(-1) + \frac{1}{4}(1) = 0$

$n_{y_1, s_1}(D(\phi)) = \frac{1}{4}(\text{mult. } y_1) + \frac{1}{4}(\text{mult. } s_1)$

$= \frac{1}{4}(-1) + \frac{1}{4}(-3)$

$= -1$

$\mu(\phi) = 0 + 0 - 1$

What about ϕ^{-1} ??? $\quad \mu(\phi^{-1}) = 1$

Prop. Neither ϕ nor ϕ^{-1} has any holomorphic representatives.

Pf. Not all multiplicities are ≥ 0 for either ϕ or ϕ^{-1}.

Let $m_{\bullet,\circ}$ denote the coefficient of $\circ$ in $\partial\bullet$.

i.e. $\displaystyle\sum_{\substack{\phi \in \pi_2(\bullet,\circ) = \cancel{\mathbb{Z}} \oplus H_2(Y) \\ n_z(\phi) = 0 \\ \mu(\phi) = 1}} \# \widehat{\mathcal{M}}(\phi)$

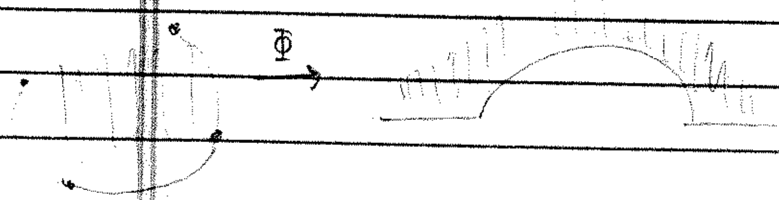
Exercise: (1) Compute the effect of splicing $a \in H_2(Y) = \mathbb{Z}\langle a \rangle$ to $\phi \in \pi_2(y_1 s_1, x_1 r_3)$

i.e. What is $\mu(\phi * a)$?

(2) Consider the effect of splicing a to constant $\in \pi_2(\vec{x}, \vec{x})$ for any $\vec{x}$. Show that this effect depends on $\vec{x}$.

Next time: • Spin^c structures

• J-holomorphic representatives.



Prop. Any Whitney disk whose domain is a rectangle (with multiplicity 1) admits a unique J-holomorphic representative.

"Pf."

$$\left\{ \begin{array}{c} F^2 \xrightarrow{\Phi} \Sigma \\ \downarrow p \\ \mathbb{D}^2 \end{array} \right\} \longleftrightarrow \left\{ \mathbb{D}^2 \xrightarrow[\text{Whitney}]{} \mathrm{Sym}^g(\Sigma) \right\}$$

Suppose we have a J-holomorphic representative:

$$\begin{array}{ccccc} \hat{F} & & (\Sigma, j) \times \cdots \times (\Sigma, j) & \to & (\Sigma, j) \\ \downarrow u^*\pi & & \downarrow \pi & & \\ F \to \mathbb{D} & \xrightarrow{u} & (\mathrm{Sym}^g(\Sigma), \mathrm{Sym}(j)) & & \end{array}$$

$u^*\pi / S_{g-1}$

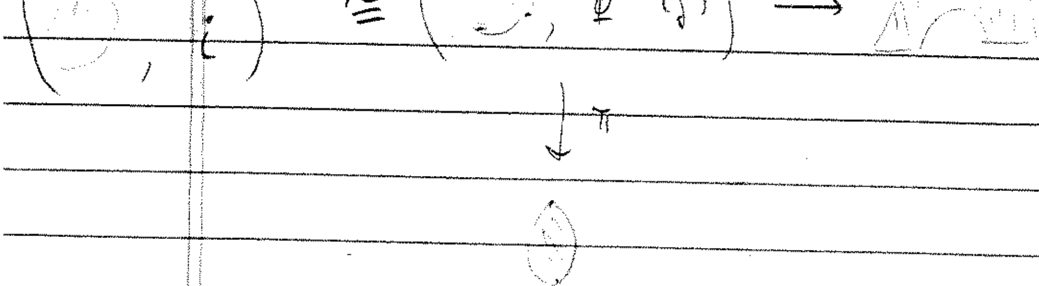
Φ

$J \circ u = u \circ i$

So p and Φ must be holomorphic as well.

Conversely, if p and Φ are holomorphic, they correspond to a holomorphic Whitney disk.

Riemann mapping Thm.



$\cong$ $\Phi^{-1}(j)$ $\xrightarrow{\Phi}$ $\downarrow \pi$