

# Principles and Methods of Capital Allocation for Enterprise Risk Management

## Lecture 4 of 4-part series

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# Outline



## Solvency defined

Solvency status of a company is assessed at a particular period requiring sufficient capital is held to cover expected liabilities over a fixed time horizon, with a high degree of probability confidence.

Technically, if $S$ is the aggregated random loss over the time horizon, the solvency capital requirement (SCR), term used in Sandström (2011), is

$$\mathrm{SCR}_S = \rho[S] - \mathbb{E}[S],$$

where $\rho$ is a risk measure defined to be a mapping from set $\Gamma$ of real-valued random variables defined on a probability space $(\Omega, \mathcal{F}, \mathbb{P})$ to the real line $\mathbb{R}$:

$$\rho : \Gamma \to \mathbb{R} : S \in \Gamma \to \rho[S].$$

Risk measures - Artzner (1999).



## The aggregation of risks

The company's aggregate loss $S$ is usually the sum of several components

$$S = X_1 + X_2 + \cdots + X_n,$$

where the components $X_1, X_2, \ldots, X_n$ can be interpreted as:

- the individual losses corresponding to the losses of the several business units within the company;
- the individual losses arising from the different policies within the company's portfolio of policies; or
- the individual losses arising from various categories of risks such as the underwriting, credit, market and operational risks.



# Popular risk measures

Premium principles are clear examples of risk measures. Goovaerts (1984).

Risk measures must be practically simple to calculate and easily understood.

Two widely known and used risk measures are:

- Value-at-Risk (VaR): For $0 < p < 1$, the $p$-th quantile risk measure is defined to be

$$\mathrm{VaR}_p[S] = \inf(s | F_S(s) \geq q).$$

- Tail Value-at-risk: The Tail VaR is defined to be

$$\mathrm{TVaR}_p[S] = \mathbb{E}(S | S > \mathrm{VaR}_p[S]).$$

Both risk measures are used in several regulatory regimes as well as by rating agencies such as Standard & Poor's.



## Possible effect of risk interactions

To determine solvency capital, convention is:

- first identify various sources of risks;
- quantify these risks (with probabilistic models);
- determine separate amount of capital needed for each risk; and
- account for possible interaction of risks which may lead to possible diversification effect.

Typically, diversification is interpreted so that this leads to some form of a benefit:

$$\mathrm{SCR}_S \leq \mathrm{SCR}_{X_1} + \cdots + \mathrm{SCR}_{X_n}.$$

Because expectation is a linear operator, this leads us to a choice of a subadditive risk measure:

$$\rho[S] \leq \rho[X_1] + \cdots + \rho[X_n].$$



# The classification of risks

A typical insurer would classify risks according to:

- Asset default risk - potential losses arising from investment default.
- Interest rate risk - risk of losses because of changes in the level of interest rates causing a mismatch in asset and liability cash flows.
- Credit risk - risk arising from inability to recover from reinsurers or other sources of risk transfer arrangements.
- Underwriting risk - risk of losses arising from excess claims (pure random fluctuations or prediction inaccuracies).
- Other business risk - the “catch-all-else” category including e.g. operational losses.



# Risk-based capital models

Most risk-based capital (RBC) models attempt to quantify capital requirements according to the company's exposure to risks.

- These are formula-based in the sense that for each sources of "quantifiable" risk, a set of factors (or percentages) are recommended to establish a set of Minimum Capital Requirements.
- This approach has been recommended by the National Association of Insurance Commissioners (NAIC) in the United States since the 1990's, and has been the model followed even till today.
- The NAIC formula-based capital requirement has been similarly adopted by rating agencies such as:
  - Standard & Poor's; and
  - A.M. Best.



# Comparing risk-based capital charges

## The case of general insurers

| Risk categories | NAIC | S & P | A.M. Best |
|---|---|---|---|
| **Asset risk charges:** | | | |
| Bonds | 0 - 30% | 0 - 30% | 0 - 30% |
| Common Stock | 20 - 43% | 15% | 15% |
| Real Estate | 18 - 29% | 10% | 20% |
| **Credit risk charges:** | | vary by | vary by |
| Reinsurance recoverables | 10% | reinsurer's rating | reinsurer's rating |
| **Written premium risk charges:** | | | |
| Homeowners | vary by line of | 21 - 35% | 37 - 54% |
| Other liability occurrence | business with initial | 30 - 49% | 32 - 40% |
| CMP | industry factor | 13 - 21% | 29 - 37% |
| Personal auto | adjusted for company | 9 - 14% | 25 - 40% |
| Property | experience | 9 - 14% | 33 - 51% |
| **Reserve risk charges:** | | | |
| Homeowners | vary by line of | 11 - 19% | 19 - 39% |
| Other liability occurrence | business with initial | 14 - 23% | 26 - 48% |
| CMP | industry factor | 5 - 9% | 25 - 45% |
| Personal auto | adjusted for company | 10 - 16% | 20 - 48% |
| Property | experience | 28 - 46% | 26 - 47% |

Source: M. Carrier, Deloitte Consulting LLP, Risk-Based Capital: So Many Models, slides at the CAS Annual Meeting 2007.



# Solvency II

Solvency II is a by-product of the European Commission to develop new solvency system of regulatory requirements for insurers to operate in the European Union.

- Framework somewhat patterned after the New Basel Capital Accord (Basel II) on banking supervision.
- To achieve some sort of uniformity in regulations for establishing capital.
- Based on broad "risk-based" principles in the measurement of assets and liabilities.
- The primary aims are:
  - to reduce the probability of insolvency; and
  - if it does occur, to reduce the financial and economic impact to affected policyholders.



# The Solvency II framework

Solvency II framework consists of 3 pillars.

- Pillar 1 - consists of identifying the risks and quantifying the amount of capital required.
  - fair valuation of assets/liabilities;
  - some prescription of factor-based methods to calculate minimum capital; but
  - use of internal models allowed, provided justified.
- Pillar 2 - prescribes requirement for effective risk management systems and processes with steps towards effective supervisory review and examination.
- Pillar 3 - focuses on a more discipline in the market including fair disclosure and more transparency.

Additional details can be found in: www.fsa.gov.uk



## Implementation of Solvency II

It appears that complete requirements be met by companies on 1 January 2014.

This means that for companies:

- even prior to full implementation, getting ready to follow procedures and be in compliance require work (maybe as early as 2013)
- need to gather data and use them to evaluate, assess, validate risks they are facing
- one possible key challenge faced by insurers is seeking for the approval of regulators to use internal models (internal models must be well justified)
- implementation obviously involves additional cost to the company both direct and indirect (e.g. administrative, interruption)

Additional details can be found in:

`http://ec.europa.eu/internal_market/insurance/`



## Approaches to aggregating risks

The aggregation of risks is the complete opposite of capital allocation.

- Standard methodology - based on the following assumptions:
  - (i) $\mathbf{X}^T = (X_1, \ldots, X_n)$ follows a multivariate normal with mean $\mu^T = (\mu_1, \ldots, \mu_n)$ and covariance $\Sigma = (\sigma_{ij})$; and
  - (ii) The risk measure used is the quantile risk measure or VaR.
- Extension to the standard methodology - based on the following assumptions:
  - (i) Each $X_i$ belongs to a location-scale family of distributions:

$$X_i = \mu_i + \sigma_i Y, \quad \text{for } i = 1, \ldots, n.$$

  - (ii) $S$ also belongs to same location-scale family: $S = \mu_S + \sigma_S Y$; and
  - (iii) Risk measure used is conditional tail expectation or TVaR.
- Numerical simulations with copulas.



# The standard methodology

$S$ has a normal distribution with mean $\mathbb{E}[S] = \sum_{i=1}^{n} \mu_i$ and variance $\text{Var}[S] = \mathbf{1}^T \Sigma \mathbf{1}$, where $\mathbf{1}^T = (1, 1, \ldots, 1)$.

Thus, we have

$$\text{SCR}_S = \text{VaR}_p[S] - \mathbb{E}[S],$$

where, using the property of normal distribution, we have

$$\text{VaR}_p[S] = \Phi^{-1}(p)\sigma_S + \mathbb{E}[S],$$

and hence,

$$\text{SCR}_S = \Phi^{-1}(p)\sigma_S = \Phi^{-1}(p)\sqrt{\text{Var}[S]} = \Phi^{-1}(p)\sqrt{\mathbf{1}^T \Sigma \mathbf{1}}.$$

$\Phi^{-1}$ denotes the quantile function of a standard normal and $\sigma_S$ is the standard deviation of $S$.



## - continued

Note that

$$\begin{aligned}
\mathbf{1}^T\Sigma\mathbf{1} &= \sum_{i=1}^{n}\sum_{j=1}^{n}\text{Cov}(X_i, X_j) = \sum_{i=1}^{n}\sum_{j=1}^{n}\sigma_i\sigma_j\rho_{ij} \\
&= \frac{1}{[\Phi^{-1}(p)]^2}\sum_{i=1}^{n}\sum_{j=1}^{n}\text{SCR}_i\text{SCR}_j\rho_{ij} = \frac{\mathbf{SCR}^T\,\Sigma\,\mathbf{SCR}}{[\Phi^{-1}(p)]^2},
\end{aligned}$$

where

$$\mathbf{SCR}^T = (\text{SCR}_{X_1}, \ldots, \text{SCR}_{X_n}),$$

the vector of stand-alone solvency capitals $\text{SCR}_{X_i}$ for each risk $i$.

This proof has appeared in Dhaene (2005). It immediately follows that

$$\text{SCR}_S = \sqrt{\mathbf{SCR}^T\,\Sigma\,\mathbf{SCR}}.$$

The stand-alone capitals can indeed be written as

$$\text{SCR}_{X_i} = \Phi^{-1}(p)\sigma_{X_i} = \Phi^{-1}(p)\sqrt{\text{Var}[X_i]}.$$



# Extension to the standard methodology

For stand-alone losses $X_i$, we have

$$\begin{aligned} \mathrm{TVaR}_p(X_i) &= \mathbb{E}[X_i | X_i > \mathrm{VaR}_p[X_i]] \\ &= \mu_i + \sigma_i \mathbb{E}[Z | Z > \mathrm{VaR}_p[Z]]) \\ &= \mu_i + \sigma_i \mathrm{TVaR}_p[Z]. \end{aligned}$$

Similarly, we have $\mathrm{TVaR}_p[S] = \mu_S + \sigma_S \mathrm{TVaR}_p[Z]$.

From here, we find that

$$\begin{aligned} \mathbf{1}^T \Sigma \mathbf{1} &= \frac{\sum_{i=1}^n \sum_{j=1}^n (\mathrm{TVaR}_p[X_i] - \mu_i) \rho_{ij} (\mathrm{TVaR}_p[X_j] - \mu_j)}{[\mathrm{TVaR}_p(Z)]^2} \\ &= \frac{1}{[\mathrm{TVaR}_p(Z)]^2} (\mathrm{TVaR}_p[\mathbf{X}] - \mu)^T \Sigma (\mathrm{TVaR}_p[\mathbf{X}] - \mu). \end{aligned}$$

where $\mathrm{TVaR}_p[\mathbf{X}] = (\mathrm{TVaR}_p[X_1], \ldots, \mathrm{TVaR}_p[X_n])^T$, the vector of stand-alone solvency capitals $\mathrm{TVaR}_p[X_i]$ for each risk $i$.



## - continued

It follows that

$$\mathrm{SCR}_S = \mu_S + \sqrt{(\mathrm{TVaR}_p[\mathbf{X}] - \mu)^T \, \Sigma \, (\mathrm{TVaR}_p[\mathbf{X}] - \mu)}.$$

A similar form to the standard methodology can be found in this case:

$$\mathrm{SCR}_S = \mu_S + \sqrt{\mathbf{SCR}^T \, \Sigma \, \mathbf{SCR}}.$$

Indeed, Dhaene (2005) provides a further extension to the class of distortion risk measures for which the Tail VaR is a special case.

This class of risk measures was introduced by Wang (1996).



# Selected reference

- Dhaene, J., Goovaerts, M.J., Lundin, M. and S. Vanduffel (2005). Aggregating economic capital, *Belgian Actuarial Bulletin*, 5: 14-25.
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