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Équations aux dérivées partielles/*Partial Differential Equations*

Linear boundary values of weakly quasiregular mappings

Baisheng YAN

Department of Mathematics, Michigan State University, East Lansing, MI 48824, USA
E-mail: yan@math.msu.edu

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Abstract. We use a new construction in the spirit of Gromov's convex integration to prove a rather surprising result that every affine map is the boundary value of a weakly quasiregular map in any Sobolev space with index $p < n/2$. Our method is based on constructing special Cauchy sequences by convex integration for certain vectorial Hamilton–Jacobi equations.

Valeurs aux limites linéaires des applications faiblement quasi régulières

Résumé. *Nous utilisons une nouvelle construction, dans l'esprit de l'intégration convexe de Gromov, pour prouver un résultat plutôt étonnant selon lequel toute application affine est la valeur frontière d'une application faiblement quasi régulière dans tout espace de Sobolev d'indice $p < n/2$. Notre méthode est basée sur une construction de suites de Cauchy spéciales, par intégration convexe pour certaines équations vectorielles de Hamilton–Jacobi.*

Version française abrégée

La méthode d'intégration convexe de Gromov [3] a été récemment appliquée avec succès à l'étude de l'existence des équations vectorielles de Hamilton–Jacobi de la forme $\mathrm{D}u(x) \in K$, par Müller–Šverák [6,7] (*voir* aussi [9]). Une approche différente, utilisant la méthode des catégories de Baire, a été étudiée par l'école italienne (*voir* par exemple Dacorogna–Marcellini [1,2]).

Dans cette Note nous montrons comment utiliser des idées de l'intégration convexe pour étudier les problèmes de valeurs aux limites affines pour les applications faiblement quasi régulières dans $\mathbb{R}^n$.

Rappelons (*voir* par exemple [4,5,10]) qu'une application u d'un domaine Ω de $\mathbf{R}^n$ dans $\mathbf{R}^n$ est dite *faiblement L-quasi régulière,* où $L \geqslant 1$ est une constante appelée la *dilatation* (externe) de u, si elle appartient à $\mathrm{W}^{1,p}_{\mathrm{loc}}(\Omega; \mathbf{R}^n)$ pour une certaine puissance $p \geqslant 1$ et satisfait

$$\left|\mathrm{D}u(x)\right|^n \leqslant L \det \mathrm{D}u(x) \quad \text{p.p. } x \in \Omega,$$

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où $\mathrm{D}u = (\partial u^i/\partial x_j)$ désigne la matrice gradient de u et $|\xi|$ la norme de matrice définie par $|\xi| = \max_{|h|=1} |\xi h|$. Nous appellerons applications *faiblement conforme* les applications faiblement 1-quasi régulières.

Nous notons K_L l'*ensemble L-quasi régulier* défini par

$$K_L = \{\xi \in M^{n\times n} \mid |\xi|^n \leqslant L \det \xi\}$$

(si $L = 1$, K_1 est dit *ensemble conforme*). Alors la classe des applications faiblement L-quasi régulières $u \in \mathrm{W}^{1,p}_{\mathrm{loc}}(\Omega;\mathbb{R}^n)$ peut être caractérisée par une équation spéciale de Hamilton–Jacobi :

$$\mathrm{D}u(x) \in K_L, \quad \text{p.p. } x \in \Omega.$$

Dans toute cette Note, nous supposons la dimension $n \geqslant 3$. Le résultat suivant peut être prouvé en utilisant les résultats sur *l'enveloppe p-quasiconvexe* d'ensemble K_L établi en Yan–Zhou [15]. (*Voir* Yan [13] pour les détails.)

THÉORÈME 1. – *Pour tout $L \geqslant 1$, il existe une puissance $p = p_n(L) < n$ telle que, si une application u faiblement L-quasi régulière dans $\mathrm{W}^{1,p}(\Omega;\mathbf{R}^n)$ est affine sur la frontière $\partial\Omega$, c'est-à-dire, $u|_{\partial\Omega} = \xi x + b$, on a $\xi \in K_L$.*

Le but principal de cette Note est de montrer que la situation est complètement différente pour les applications faiblement quasi régulières dans les espaces de Sobolev $\mathrm{W}^{1,p}(\Omega;\mathbf{R}^n)$ si la puissance $p < n$ n'est pas voisine de la dimension n.

THÉORÈME 2. – *Toute application affine est la valeur frontière d'une application faiblement conforme dans $\mathrm{W}^{1,p}(\Omega;\mathbf{R}^n)$, pour tout $1 \leqslant p < n/2$.*

THÉORÈME 3. – *Soit $1 \leqslant p < n/2$. Alors pour toute application affine par morceaux $\varphi \in \mathrm{W}^{1,p}(\Omega;\mathbf{R}^n)$ et pour tout $\varepsilon > 0$, il existe une application faiblement conforme $u_\varepsilon \in \varphi + \mathrm{W}^{1,p}_0(\Omega;\mathbf{R}^n)$ telle que $\|u_\varepsilon - \varphi\|_{\mathrm{L}^p(\Omega)} < \varepsilon$.*

Remarques

1. Ici et dans toute la suite, nous disons que $\varphi \in \mathrm{W}^{1,p}(\Omega;\mathbf{R}^n)$ est *affine par morceaux* s'il existe une famille au plus dénombrable de sous-ensembles disjoints Ω_j de Ω, dont l'union est de mesure pleine, telle que chaque $\varphi|_{\Omega_j}$ est affine.
2. Soit $n \geqslant 4$ un nombre pair. Alors le théorème 2 et un résultat de Iwaniec–Martin [5] (*voir* aussi Müller–Šverák–Yan [8]) impliquent que la puissance optimale p pour les applications faiblement conformes dans le théorème 1 est $p_n(1) = n/2$. On ne sait pas si $p_n(1) = n/2$ pour les dimensions n impaires (*voir* [4,11,14]).

1. Introduction

In recent years, the method of Gromov's convex integration (*see* [3]) has been successfully applied in studying the existence of vectorial Hamilton–Jacobi equations of the form $\mathrm{D}u(x) \in K$, by Müller–Šverák [6,7] (*see* also [9]). A different approach to study the existence of vectorial Hamilton–Jacobi equations using the Baire category method has been pursued by the Italian school (*see*, e.g., Dacorogna–Marcellini [1,2]).

In this Note, we show how to use some ideas in the spirit of Gromov's convex integration, to investigate the linear boundary values for weakly quasiregular mappings in $\mathbb{R}^n$.

Recall that (*cf.* [4,5,10]) a map u from a domain Ω in $\mathbf{R}^n$ to $\mathbf{R}^n$ is said to be *weakly L-quasiregular*, $L \geqslant 1$ being a constant called the (outer) *dilatation* of u, if it belongs to $\mathrm{W}^{1,p}_{\mathrm{loc}}(\Omega;\mathbf{R}^n)$ for some $p \geqslant 1$ and satisfies

$$\left|\mathrm{D}u(x)\right|^n \leqslant L \det \mathrm{D}u(x) \quad \text{a.e. } x \in \Omega,$$

where $\mathrm{D}u = (\partial u^i/\partial x_j)$ denotes the gradient matrix of u and $|\xi|$ is the matrix norm defined by $|\xi| = \max_{|h|=1} |\xi h|$. The weakly 1-quasiregular maps will be called the *weakly conformal* maps.

Denote by K_L the *L-quasiregular set* defined by

$$K_L = \left\{\xi \in M^{n\times n} \mid |\xi|^n \leqslant L \det \xi\right\}$$

(if $L = 1$, K_1 is called the conformal set). Then the class of L-weakly quasiregular mappings $u \in \mathrm{W}^{1,p}_{\mathrm{loc}}(\Omega; \mathbb{R}^n)$ can be characterized by a special Hamilton–Jacobi equation:

$$\mathrm{D}u(x) \in K_L, \quad \text{p.p. } x \in \Omega.$$

Throughout this Note, we assume the dimension $n \geqslant 3$. The following result can be proved using the results on the so-called *p-quasiconvex hull* of the set K_L established in Yan–Zhou [15]. (*See* Yan [13] for details.)

THEOREM 1. – *For each $L \geqslant 1$ there exists an index $p = p_n(L) < n$ such that if a weakly L-quasiregular map u in $\mathrm{W}^{1,p}(\Omega; \mathbf{R}^n)$ assumes an affine boundary value $u|_{\partial\Omega} = \xi x + b$, then $\xi \in K_L$.*

The main purpose of this Note is to show that situation is completely different for weakly quasiregular mappings in Sobolev spaces $\mathrm{W}^{1,p}(\Omega; \mathbf{R}^n)$ if index $p < n$ is not too close to the dimension n.

THEOREM 2. – *Every affine map is the boundary value of a weakly conformal map in $\mathrm{W}^{1,p}(\Omega; \mathbf{R}^n)$ for any given $1 \leqslant p < n/2$.*

THEOREM 3. – *If $1 \leqslant p < n/2$, then for any piece-wise affine map $\varphi \in \mathrm{W}^{1,p}(\Omega; \mathbf{R}^n)$ and $\varepsilon > 0$, there exists a weakly conformal map $u_\varepsilon \in \varphi + \mathrm{W}^{1,p}_0(\Omega; \mathbf{R}^n)$ such that $\|u_\varepsilon - \varphi\|_{\mathrm{L}^p(\Omega)} < \varepsilon$.*

Remarks

1. Here and in the following we say that $\varphi \in \mathrm{W}^{1,p}(\Omega; \mathbf{R}^n)$ is *piece-wise affine* if there exist at most countably many disjoint open subsets Ω_j of Ω whose union has full measure such that each $\varphi|_{\Omega_j}$ is affine.

2. If $n \geqslant 4$ is even, Theorem 2 and a result of Iwaniec–Martin [5] (*see* also Müller–Šverák–Yan [8]) imply that the optimal index p for weakly conformal mappings in Theorem 1 is $p_n(1) = n/2$. Whether $p_n(1) = n/2$ for odd n remains open (*see* [4,11,14]).

2. A general existence theorem

Let $M^{m\times n}$ be the space of all real $m \times n$ matrices with the norm $|\xi|$ defined as above. Let K be a subset of $M^{m\times n}$. Define the sets $\mathcal{L}_j(K)$ inductively as follows: $\mathcal{L}_0(K) = K$ and, for $j = 0, 1, \ldots,$

$$\mathcal{L}_{j+1}(K) = \left\{t\xi + (1-t)\eta \;\middle|\; t \in [0,1],\ \xi,\ \eta \in \mathcal{L}_j(K),\ \operatorname{rank}(\xi - \eta) \leqslant 1\right\}.$$

Define the *lamination hull* of K (*see*, e.g., [1,2,6,7]) to be the set

$$\mathcal{L}(K) = \bigcup_{j=0}^{\infty} \mathcal{L}_j(K).$$

The following result can be proved by using a useful lemma found in [1,9]. (*See* details in [13].)

PROPOSITION 1. – *Let $B = \mathcal{L}(A)$ be an open set in $M^{m\times n}$. Then for any $\xi \in B$ and $\varepsilon > 0$, there exist a piece-wise affine map $u \in \xi x + \mathrm{W}^{1,\infty}_0(\Omega; \mathbf{R}^m)$ and two sets of finitely many points $\{\alpha_1, \alpha_2, \ldots, \alpha_r\} \subset A$ and $\{\xi_1, \xi_2, \ldots, \xi_q\} \subset B$ such that*

$$\mathrm{D}u(x) \in \{\alpha_1, \ldots, \alpha_r\} \cup \{\xi_1, \ldots, \xi_q\}, \quad \textit{a.e. } x \in \Omega,$$

and the measure $\left|\{x \in \Omega \mid \mathrm{D}u(x) \notin \{\alpha_1, \alpha_2, \ldots, \alpha_r\}\}\right| < \varepsilon$.

For $p \geqslant 1$, let $\beta_p(K)$ be the set of all matrices ξ such that there exists a map $u = u_\xi \in \mathrm{W}^{1,p}(\Omega; \mathbf{R}^m)$ satisfying

$$\mathrm{D}u(x) \in K \quad \text{a.e. } x \in \Omega; \quad u|_{\partial\Omega} = \xi x.$$

If $\xi \in K$, we can always choose $u_\xi \equiv \xi x$; hence $K \subseteq \beta_p(K)$.

A general existence theorem which we shall use to prove the main results is the following.

THEOREM 4. – *Let* $K \subset M^{n\times m}$ *be a closed set and let* $A \subset \beta_p(K)$ *be a set satisfying*

$$c_0 = \sup_{\xi \in A} \frac{1}{|\Omega|} \int_\Omega |\mathrm{D}u_\xi|^p \, \mathrm{d}x < \infty. \tag{1}$$

Suppose the set $B = \mathcal{L}(A)$ *is open and bounded. Then* $B \subset \beta_p(K)$.

Sketch of proof. – Let $\xi \in B$ be given and let $\varepsilon_k \to 0^+$ be a decreasing sequence satisfying $\sum_k \varepsilon_k^{1/p} < \infty$. Let u be constructed as in Proposition 1 with $\varepsilon = \varepsilon_1$. Let $\Sigma = \{x \in \Omega \mid \mathrm{D}u(x) \in A\} = \bigcup_{j=1}^\infty \Sigma_j$, where $u|_{\Sigma_j} = \alpha_{i(j)} x + d_j$ is affine for some $1 \leqslant i(j) \leqslant r$. From condition (1) and by a Vitali covering argument, there exists $v_j \in u + \mathrm{W}_0^{1,p}(\Sigma_j; \mathbb{R}^m)$ such that $\mathrm{D}v_j(x) \in K$ a.e. in Σ_j and $\int_{\Sigma_j} |\mathrm{D}v_j|^p \, \mathrm{d}x \leqslant c_0 |\Sigma_j|$. Let $u_1 = \chi_{\Omega \setminus \Sigma} u + \sum_{j=1}^\infty \chi_{\Sigma_j} v_j$. Then $u_1 \in \xi x + \mathrm{W}_0^{1,p}(\Omega; \mathbb{R}^m)$ satisfies

$$\begin{aligned}
&\mathrm{D}u_1(x) \in K \cup \{\xi_1, \ldots, \xi_q\},\\
&\Omega_\varepsilon = \left\{x \in \Omega \mid \mathrm{D}u_1(x) \in \{\xi_1, \ldots, \xi_q\}\right\},\\
&|\Omega_\varepsilon| < \varepsilon, \qquad \int_{\Omega \setminus \Omega_\varepsilon} |\mathrm{D}u_1|^p \, \mathrm{d}x \leqslant c_0 |\Omega \setminus \Omega_\varepsilon|.
\end{aligned}$$

We modify the values of u_1 on each open set where $\mathrm{D}u_1 \notin K$. Let $\Omega_1 = \Omega_\varepsilon$ be the set defined in the above construction. Write $\Omega_1 = \bigcup_{j=1}^\infty \Delta_j$ such that $u_1|_{\Delta_j} = \xi_j x + d_j$ and $\xi_j \in B$. For each $j = 1, 2, \ldots,$ we use the previous construction for $\xi_j \in B$ with domain Δ_j and number $\varepsilon = \varepsilon_2/2^j$ to obtain a map $\tilde{v}_j \in u_1 + \mathrm{W}_0^{1,p}(\Delta_j; \mathbb{R}^m)$ such that

$$\begin{aligned}
&\mathrm{D}\tilde{v}_j(x) \in K \cup \{\xi_1', \ldots, \xi_{q'}'\},\\
&\Delta_j' = \left\{x \in \Delta_j \mid \mathrm{D}\tilde{v}_j(x) \in \{\xi_1', \ldots, \xi_{q'}'\}\right\},\\
&|\Delta_j'| < \varepsilon_2/2^j, \qquad \int_{\Delta_j \setminus \Delta_j'} |\mathrm{D}\tilde{v}_j|^p \, \mathrm{d}x \leqslant c_0 |\Delta_j \setminus \Delta_j'|.
\end{aligned}$$

Let $\Omega_2 = \bigcup_{j=1}^\infty \Delta_j'$. Then $|\Omega_2| < \varepsilon_2$. Define $u_2 = \chi_{\Omega \setminus \Omega_1} u_1 + \sum_{j=1}^\infty \chi_{\Delta_j} \tilde{v}_j$. Then $u_2 \in \xi x + \mathrm{W}_0^{1,p}(\Omega; \mathbb{R}^m)$ satisfies that $u_2 = u_1$ on $\Omega \setminus \Omega_1$, $\mathrm{D}u_2(x) \in B$ a.e. in Ω_2, $\mathrm{D}u_2(x) \in K$ a.e. in $\Omega \setminus \Omega_2$, and $\int_{\Omega_1 \setminus \Omega_2} |\mathrm{D}u_2|^p \, \mathrm{d}x \leqslant c_0 |\Omega_1 \setminus \Omega_2|$.

We then modify the values of u_2 on the set Ω_2 as we did for u_1 on Ω_1 to obtain u_3 and Ω_3. Continuing in this way, we obtain a sequence $\{u_k\}$ in $\xi x + \mathrm{W}_0^{1,p}(\Omega; \mathbb{R}^m)$ and open sets $\Omega_k \subset \Omega_{k-1} \subset \Omega$ such that:

$$|\Omega_k| < \varepsilon_k, \tag{2}$$

$$\mathrm{D}u_k(x) \in B \quad \text{a.e. in } \Omega_k, \tag{3}$$

$$\mathrm{D}u_k(x) \in K \quad \text{a.e. in } \Omega \setminus \Omega_k, \tag{4}$$

$$u_{k+1} = u_k \quad \text{on } \Omega \setminus \Omega_k, \tag{5}$$

$$\int_{\Omega_k \setminus \Omega_{k+1}} |\mathrm{D}u_{k+1}|^p \, \mathrm{d}x \leqslant c_0 |\Omega_k \setminus \Omega_{k+1}|. \tag{6}$$

First of all, assuming $|\eta| < \lambda$ for all $\eta \in B$, by conditions (3), (6), we have

$$\int_{\Omega_k} |\mathrm{D}u_{k+1}|^p \, \mathrm{d}x \leqslant c_0 \, |\Omega_k \setminus \Omega_{k+1}| + \lambda^p \, |\Omega_{k+1}| \leqslant C_0 \, |\Omega_k|,$$

where $C_0 = \max\{c_0, \lambda^p\}$. Hence, by (5),

$$\|\mathrm{D}u_{k+1} - \mathrm{D}u_k\|_{\mathrm{L}^p(\Omega)} = \|\mathrm{D}u_{k+1} - \mathrm{D}u_k\|_{\mathrm{L}^p(\Omega_k)} \leqslant 2\, C_0^{1/p} \, |\Omega_k|^{1/p}. \tag{7}$$

Furthermore, by (2)–(6), we easily obtain that:

$$\int_{\Omega} |\mathrm{D}u_{k+1}|^p \, \mathrm{d}x \leqslant C_0 \, |\Omega|, \tag{8}$$

$$\int_{\Omega} \operatorname{dist}^p(\mathrm{D}u_k(x); K) \, \mathrm{d}x \leqslant C' \, |\Omega_k| < C' \, \varepsilon_k, \tag{9}$$

where C' is a constant and $\operatorname{dist}(\eta; K)$ is the distance function to the set K.

Finally, conditions (7), (8) and the convergence of $\sum_k \varepsilon_k^{1/p}$ imply that the sequence $\{u_k\}$ is a Cauchy sequence in $\xi x + \mathrm{W}_0^{1,p}(\Omega; \mathbb{R}^m)$. Let $\bar{u} \in \xi x + \mathrm{W}_0^{1,p}(\Omega; \mathbb{R}^m)$ be the limit of this sequence. Since K is closed, condition (9) implies $\mathrm{D}\bar{u}(x) \in K$ for almost every $x \in \Omega$. This proves $\xi \in \beta_p(K)$; hence, $B \subset \beta_p(K)$.

The proof of Theorem 4 is now completed. □

Remark. – From (8) in the proof above we easily see that the solution $\bar{u} \in \xi x + \mathrm{W}_0^{1,p}(\Omega; \mathbb{R}^m)$ obtained in the proof also satisfies

$$\frac{1}{|\Omega|} \int_{\Omega} |\mathrm{D}\bar{u}|^p \, \mathrm{d}x \leqslant C_0 \equiv \max\Big\{ c_0, \sup_{\eta \in B} |\eta|^p \Big\}.$$

3. Boundary values of weakly conformal maps

In this section, we prove our main results: Theorems 2 and 3. To apply the general existence theorem proved above, we introduce some notations. Let for $\lambda > 0$,

$$\begin{aligned} R(n) &= \big\{ \xi \in M^{n \times n} \mid |\xi|^n = |\det \xi| \big\}, \\ A_\lambda &= \big\{ \xi \in R(n) \mid |\xi| < \lambda \big\}, \\ B_\lambda &= \big\{ \xi \in M^{n \times n} \mid |\xi| < \lambda \big\}. \end{aligned}$$

It is easy to see, by Hadamard's inequality, that the conformal set $K_1 = \{\xi \in R(n) \mid \det \xi \geqslant 0\}$. From a refinement of the arguments in Yan [11,12], we can prove the following results (*see* [13]).

PROPOSITION 2. – *We have $\mathcal{L}_{n-1}(A_\lambda) = B_\lambda$. Hence $\mathcal{L}(A_\lambda) = B_\lambda$ for all $\lambda > 0$.*

PROPOSITION 3. – *Let $1 \leqslant p < n/2$ and let $\mathbf{B}$ be the open unit ball in $\mathbb{R}^n$. Then for any $\xi \in R(n)$ there exists a weakly conformal map $u_\xi \in \xi x + \mathrm{W}_0^{1,p}(\mathbf{B}; \mathbb{R}^n)$ such that $\int_{\mathbf{B}} |\mathrm{D}u_\xi|^p \, \mathrm{d}x \leqslant C_{n,p} \, |\xi|^p$, where $C_{n,p}$ is a constant depending only on n and p.*

Proof of Theorem 2. – Let $\xi \in M^{n \times n}$ be given. We assume $\xi \neq 0$. Let $\lambda = 2|\xi|$. Then $\xi \in B_\lambda$. From Proposition 2, $\mathcal{L}(A_\lambda) = B_\lambda$ is open and bounded; one also has $\sup_{\eta \in B_\lambda} |\eta|^p \leqslant C_1 \, |\xi|^p$. Also, from Proposition 3, the set $A = A_\lambda \subset \beta_p(K_1)$ satisfies the condition (1) in Theorem 4 with constant $c_0 \leqslant C_2 \, |\xi|^p$. Therefore, Theorem 4 implies $\xi \in \beta_p(K_1)$. This proves the theorem. □

From the remark following the proof of Theorem 4 and by a Vitali covering argument, we easily obtain the following result from Theorem 2.

COROLLARY 1. – *For any* $\xi \in M^{n\times n}$, $b \in \mathbf{R}^n$ *and* $\varepsilon > 0$, *there exists a weakly conformal map* $u_\varepsilon \in \xi x + b + \mathrm{W}_0^{1,p}(\Omega;\mathbb{R}^n)$ *satisfying*

$$\int_\Omega |\mathrm{D}u_\varepsilon|^p \,\mathrm{d}x \leqslant C_3\,|\xi|^p\,|\Omega|, \qquad \|u_\varepsilon - \xi x - b\|_{\mathrm{L}^p(\Omega)} < \varepsilon,$$

where C_3 *is a constant depending only on* n *and* p.

Proof of Theorem 3. – Let $\varphi \in \mathrm{W}^{1,p}(\Omega;\mathbb{R}^n)$ be a piece-wise affine map. Let $\Omega = \bigcup_{j=1}^\infty \Omega_j \cup N$, $|N| = 0$, be such that $\varphi|_{\Omega_j} = \xi_j x + b_j$ is affine for all $j \geqslant 1$. The condition $\varphi \in \mathrm{W}^{1,p}(\Omega;\mathbb{R}^n)$ implies $\sum_j |\xi_j|^p\,|\Omega_j| < \infty$. We apply Corollary 1 to each ξ_j, b_j and Ω_j and obtain weakly conformal maps $u_j \in \varphi + \mathrm{W}_0^{1,p}(\Omega_j;\mathbb{R}^n)$ satisfying

$$\int_{\Omega_j} |\mathrm{D}u_j|^p \,\mathrm{d}x \leqslant C_3\,|\xi_j|^p\,|\Omega_j|, \qquad \|u_j - \varphi\|_{\mathrm{L}^p(\Omega_j)} < \frac{\varepsilon}{2^{j/p}}.$$

Then it is easily seen that the map $u = \sum_j \chi_{\Omega_j} u_j$ belongs to $\varphi + \mathrm{W}_0^{1,p}(\Omega;\mathbb{R}^n)$, is weakly conformal and satisfies $\|u - \varphi\|_{\mathrm{L}^p(\Omega)} < \varepsilon$. The proof is completed. □

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