

The Retirement Interest Dispersion Company Data Analysis

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June 22, 2010

Abstract At the end of every quarter the Retirement Interest Dispersion Company (RIDC) of Michigan allocates out investment gains and losses to over 2,000 individual entities based on their average daily balance for that quarter. The organization's Oracle software executes this function. RIDC has performed spot checks to ensure the Oracle software works correctly and confirms that the dispersions are correct. They have requested that a manual check of the amounts be performed concerning the fourth fiscal quarter of 2009. This report provides the results of this independent data analysis.

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Nomenclature (In Order of Appearance)

D_k	k^{th} Dollar Difference between MSU and RIDC Allocations
T_k	Sum of all Interest Subtotals for the k^{th} Department
M_k	RIDC Interest Allocation for the k^{th} Department
P_k	k^{th} Percentage Difference between MSU and RIDC Allocations
F_j	j^{th} Frequency of Occurrence for Percentage Differences between MSU and RIDC Allocations
R_j	j^{th} Range of Percentage Differences between MSU and RIDC Allocations
a_i	i^{th} Account Transaction
d_i	i^{th} Date of Account Transaction
c_i	i^{th} Date Conversion
x_i	i^{th} Reduction Factor
y_i	i^{th} Adjusted Amount
z_i	i^{th} Interest Subtotal
t	Total Amount Invested with the RIDC in Reviewed Quarter (All Departments)
I	Total Interest Dispersed by RIDC in Reviewed quarter (All Departments)

Introduction

The Retirement Interest Dispersion Company (RIDC) of Michigan is a non-profit public corporation that provides retirement plan and related benefit programs for retirees in the state of Michigan. It offers employee benefit programs: defined benefit, defined contribution and hybrid plans, and group insurance products. The organization serves cities, counties, hospitals, libraries, medical care facilities, road commissions, townships, villages, and similar units of local government [1].

RIDC allocates investment gains and losses to over 2,000 individual entities based on their average daily balances at the end of every quarter. They use Oracle software to do the allocations. Occasionally internal spot checks are performed on the allocations to ensure that the Oracle software is dispersing the gains and losses correctly to each entity. The results of the spot checks indicate that the Oracle software is performing adequately.

RIDC has requested that a manual check be performed to confirm that the dispersions are correct. This study will review the database of over 2,000 departments, calculate the interest dispersions, and check whether or not the Oracle software is correctly allocating the investment gains and losses.

This report will begin by providing a description of the supplied raw data followed by the procedure used by RIDC to allocate interest according to the amount invested and the dates for each department. Following the procedure will be the analysis describing the set of goals and basic ideas that need to be established. After calculating the interest dispersions, comparisons will be made to the interest allocation figures provided. This will lead to a discussion of the results, followed by the final conclusions of this study.

RIDC Data

The data for this project is recorded from the fourth fiscal quarter of 2009 (July 1 through September 30). There are a total of 2,083 departments recorded in the data. The first four digits for each department refer to the municipality in question, while the last four digits refer to a specific entity within the municipality, such as the fire department, court staff, or others.

The data was supplied with the full amounts for the transactions. Table 1 (seen on page 3) displays the raw data for department 01010101. The first column represents the account transaction numbers. Note that Beg Bal corresponds to the beginning balance for the selected department in the reviewed quarter. The second column displays the dollar amounts for each transaction while the third column displays the dates associated with each transaction.

Table 1. Original Data for Department 01010101.

Account	Amount (\$)	Date
Beg Bal	301,309.4	7/1/09
124100	1,604.03	7/6/09
401100	-269.35	7/17/09
124100	1,466.09	7/20/09
301100	1,466.65	7/31/09
124100	-1,466.65	7/31/09
401100	-269.35	8/18/09
124100	1,466.65	8/18/09
301100	1,467.77	8/31/09
124100	-1,467.77	8/31/09
124100	1,467.77	9/15/09
401100	-269.35	9/18/09
301100	1,468.33	9/30/09
124100	-1,468.33	9/30/09

In order to determine the correct interest allocations, RIDC calculates an average daily balance for each department and disperses payments based on those balances. This process is explained in the following section.

Along with the provided raw data, there is a list of the recorded interest distributions during this quarter for each department. This list is provided to compare the results obtained independently by this study and the dispersions provided by RIDC. An excerpt of the data displaying the interest dispersions by RIDC is shown in Table 2. The third column represents the dispersion from RIDC to the department listed in the second column. Note that the account transaction numbers are the same for each department in Table 2. This is the interest dispersion transaction number used by RIDC.

Table 2. Interest Dispersions Provided by RIDC.

Account	Department	Amount (\$)
233000	01010101	27,402.60
233000	01010102	138,181.17
233000	01010111	26,108.49
233000	01010112	108,560.74
233000	01010113	35,128.33
233000	01010114	63,145.88
233000	01010120	52,254.40
233000	01010121	38,648.99
233000	02010101	440,617.48

RIDC Calculation Process

To determine the correct interest dispersions, RIDC calculates an average daily balance for each department and distributes the gains and losses proportionally to this average daily balance.

RIDC calculates their quarterly interest dispersions in the following manner. First the transaction dates (as seen in Table 1) are converted to general numbers. For example, July 1, 2009 (the beginning of the quarter) is represented as the value 39,995 and September 30, 2009 (the end of the quarter) is represented as 40,086.

Next, conversion values are calculated. The conversion values represent how long each account transaction is from the end of the quarter. Thus, the transaction number 124100 occurring on the date 40,000 (July 6, 2009) for department 01010101 is $40,087 - 40,000 = 87$ days from the end of the quarter.

After the conversion values are calculated factors are derived to provide an adjusted amount for the interest dispersions. The factors are the conversion values for each account transaction divided by the total number of days in the quarter (*i.e.* $87/92 = 0.945652$).

These factors are then multiplied by the amounts for each account transaction and yield the adjusted amounts. The final interest dispersions are the adjusted amounts divided by the total amount invested, for all departments, with RIDC in the reviewed quarter then multiplied by the total interest, for all departments, dispersed in the same quarter. The steps listed above can be seen in Table 6 (see Appendix I).

The results of these calculations are seen in Table 3. The last column displays this study's calculated distribution values. Note that Tables 3 and 6 are the calculations and resulting values for department 01010101 in the database. Observe that the first three columns of Table 3 are the values from the raw data (see Table 1) with the dates converted to general numbers. Note that the factor values contain a six decimal accuracy whereas the dollar amounts contain a two decimal accuracy. This is done to stay consistent with the procedure used by RIDC to calculate their distributions.

Table 3. Table of Calculated Values for Department 01010101.

Account	Amount (\$)	Date	Conversion	Factor	Adjusted Amount (\$)	Interest Subtotal (\$)
Beg Bal	301,309.40	39,995	92.00	1.000000	301,309.41	27,737.92
124100	1,604.03	40,000	87.00	0.945652	1,516.85	139.64
401100	-269.35	40,011	76.00	0.826087	-222.51	-20.48
124100	1,466.09	40,014	73.00	0.793478	1,163.31	107.09
301100	1,466.65	40,025	62.00	0.673913	988.39	90.99
124100	-1,466.65	40,025	62.00	0.673913	-988.39	-90.99
401100	-269.35	40,043	44.00	0.478261	-128.82	-11.86
124100	1,466.65	40,043	44.00	0.478261	701.44	64.57
301100	1,467.77	40,056	31.00	0.336957	494.57	45.53
124100	-1,467.77	40,056	31.00	0.336957	-494.57	-45.53
124100	1,467.77	40,071	16.00	0.173913	255.26	23.50
401100	-269.35	40,074	13.00	0.141304	-38.06	-3.50
301100	1,468.33	40,086	1.00	0.010870	15.96	1.47
124100	-1,468.33	40,086	1.00	0.010870	-15.96	-1.47
Sum of Subtotals (\$)						28,036.88

Data Analysis

There are a multitude of methods for processing and organizing large quantities of data. Before processing the data, though, there are three common steps that must be considered for an effective plan. The first is to specify the objective of the study, followed by identifying the variables of interest, then choosing an appropriate design for the study [2]. These steps are explained in more detail below. After choosing a functional strategy, the data can be organized to facilitate the analysis process. Common and recommended methods of organization include grouping the data into classes and illustrating results with histograms [3]. These strategies are employed herein.

I. Analytical Metrics

Properly examining such an expansive data set requires an explanation of the metrics used.

The most important task laid out is to determine those departments where the calculated dispersions in this study are significantly different than the values provided by RIDC. Thus a metric must be created that displays the total dollar difference between the two amounts for each department. Those departments that display large differences will be identified and questioned.

Calculating the total dollar difference will not be sufficient though. The dispersions recorded by RIDC range from single digit dollar amounts to seven digit dollar amounts. For example, department 11070101 has a recorded distribution of \$3.64, whereas department 82160101 has a recorded distribution of \$4,440,185.18. Simply listing the largest total dollar differences between the calculated amounts and RIDC's values will hide those departments where there is a significant difference but not sufficiently large to be easily pinpointed for questioning.

Instead, contrasting the values using a percentage eliminates the dollar units and allows for the comparisons of all departments regardless of the sizes of the dispersions. Thus it is possible to determine those departments that may not have large dollar differences, but do have large percentage differences.

For example, recall department 11070101 has a recorded distribution of \$3.64, but this study calculated its dispersion to be \$40.44. Thus the calculated value is 1,010.99% larger than the dispersion recorded by RIDC, but the dollar difference is only \$36.80. On the other hand, department 82160101 has a recorded distribution of \$4,440,185.18 whereas the calculated value is \$4,439,168.95. This calculated value is 0.02% smaller than RIDC's value, but the dollar difference is \$1,016.23.

Using percentages allows for an easy grouping of the differences of comparisons into certain ranges. Comparisons that display a percentage above or below a certain threshold can be identified and questioned.

II. Analytical Process

The data was first split to look at each department separately, thus creating separate classes. Following the process RIDC uses for allocating interest as explained above, calculations were done for each department based on the data supplied from RIDC. After finishing these calculations a spreadsheet was created to compare the completed calculations against the provided values from RIDC. An excerpt of this spreadsheet is seen as Table 4 in the Results section.

As explained above, two measures were created to better compare the values provided by RIDC and the calculated dispersions. First, based on each pair of compared values, the dollar difference, D_k , was measured as $D_k = T_k - M_k$, where T_k is the sum of calculated interest subtotals and M_k is the RIDC interest allocation for the k^{th} department. Second, a ratio, P_k , was created to display the percentage difference between each pair of compared values. The formula for the ratio is $P_k = \left(\frac{T_k}{M_k} - 1 \right) \times 100$, where T_k and M_k are the same as above.

After calculating the percentage differences for all of the departments, the results were organized in a fashion to display a frequency counter that tallies the amount of comparisons that fall into various percentage ranges. For example, there are a certain

number of departments where the percentage difference between the calculated value and the dispersion recorded by RIDC is between 0.00% and 1.00%. The ranges used and the numbers of departments that fall into each range are seen in Table 5 in the Results section. Note that positive percentages correspond to calculated dispersions that are greater than the allocation figures provided by RIDC.

After organizing the data in this fashion, the departments that exhibited a percentage difference outside of the acceptable range were submitted to RIDC for further investigation. RIDC requested a list of those departments displaying percentage differences that were greater than 5.00% and less than -5.00%. A list of these departments is seen in Appendix II.

III. Analysis Workflow

In order to better understand the analytical process explained above, Figure 1 displays a workflow to visually display the steps used to analyze the data from RIDC.

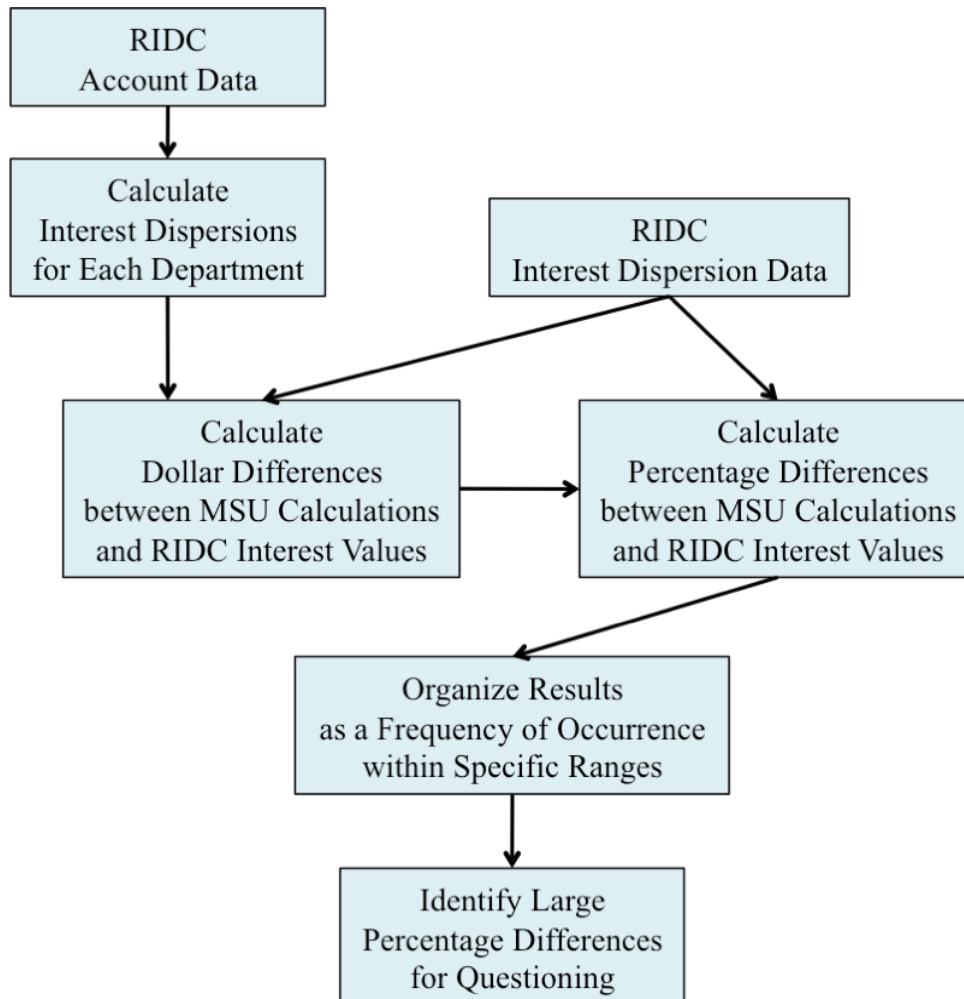


Figure 1. The Data Analysis Process Workflow.

Results

Following the distribution method employed by RIDC, calculations were completed for each department in the data set. Table 4 displays a selection of the obtained results. The first column of the table represents the departments. The second column displays this study's interest allocations against RIDC's recorded allocations seen in column three. The fourth column shows the dollar differences between the two allocations, and the last column displays the percentage difference between the allocations. The last row of the table displays the formulas and sources of values used. Note that positive dollar and percentage differences claim that the calculated dispersions are larger than the recorded dispersions.

Table 4. MSU Distributions Compared to RIDC Distributions.

Department	MSU Allocation (\$) (T_k)	RIDC Allocation (\$) (M_k)	Difference (\$) (D_k)	Difference (%) (P_k)
01010101	28,036.88	27,402.60	634.28	2.31
01010102	138,333.53	138,181.17	152.36	0.11
01010111	29,559.28	26,108.49	3,450.79	13.22
01010112	109,647.78	108,560.74	1,087.04	1.00
01010113	35,098.02	35,128.33	-30.31	-0.09
01010114	63,262.90	63,145.88	117.02	0.19
01010120	52,286.35	52,254.40	31.95	0.06
01010121	38,882.55	38,648.99	233.56	0.60
02010101	439,752.65	440,617.48	-864.83	-0.20
02020101	246,681.80	247,620.30	-938.50	-0.38
02020102	52,384.68	52,583.98	-199.30	-0.38
02020105	2,464.14	2,473.51	-9.37	-0.38
02020110	44,618.18	44,787.93	-169.75	-0.38
02030101	4,453.74	4,426.42	27.32	0.62
02030102	180,604.93	179,988.43	616.50	0.34
T_k = Sum of all Interest Subtotals for the k^{th} Department M_k = RIDC Interest Allocation for the k^{th} Department $D_k = T_k - M_k$ $P_k = (T_k / M_k - 1) \times 100$				

As explained in the analysis, the departments were organized in a manner to display the number of percentage differences that fall into various percentage ranges. For example, there are 734 departments where the percentage differences between this study's calculations and the recorded dispersions by RIDC are between 0.00% and 1.00%. These ranges and the corresponding frequencies are seen in Table 5. From this table it is seen that there are 43 of the 2,083 departments that fall outside of the -5.00% to 5.00% range that were identified for questioning.

Table 5. Frequency Counter.

Frequency (F_j)	Range (R_j)	
4	-44.07%	-9.00%
0	-9.00%	-8.00%
1	-8.00%	-7.00%
0	-7.00%	-6.00%
1	-6.00%	-5.00%
4	-5.00%	-4.00%
3	-4.00%	-3.00%
3	-3.00%	-2.00%
4	-2.00%	-1.00%
1197	-1.00%	0.00%
734	0.00%	1.00%
53	1.00%	2.00%
30	2.00%	3.00%
10	3.00%	4.00%
2	4.00%	5.00%
5	5.00%	6.00%
4	6.00%	7.00%
2	7.00%	8.00%
4	8.00%	9.00%
22	9.00%	1,010.99%
$F_j = j^{\text{th}}$ Frequency of Occurance $R_j = j^{\text{th}}$ Range of Percentage Differences		

A histogram displaying the data in Table 5 is presented in Figure 2. Notice that the horizontal axis of the histogram is not constant. This was done to provide more detail pertaining to the number of frequencies between the ranges of -1.00% and 1.00% as the vast majority of the percentage differences lie in this range.

Observe in Figure 2 there is a large frequency of percentage differences that occurs in the -0.30% to -0.40% range. This may be due to a slight calculation difference employed by RIDC and this study. The Oracle software RIDC uses for the dispersions may carry the calculations to a greater accuracy thus creating a -0.30% to -0.40% disparity between all comparisons. As the percentage differences are only three to four tenths of a percent, and the acceptable range is between -5.00% and 5.00%, the occurrence is not of high importance to this study.

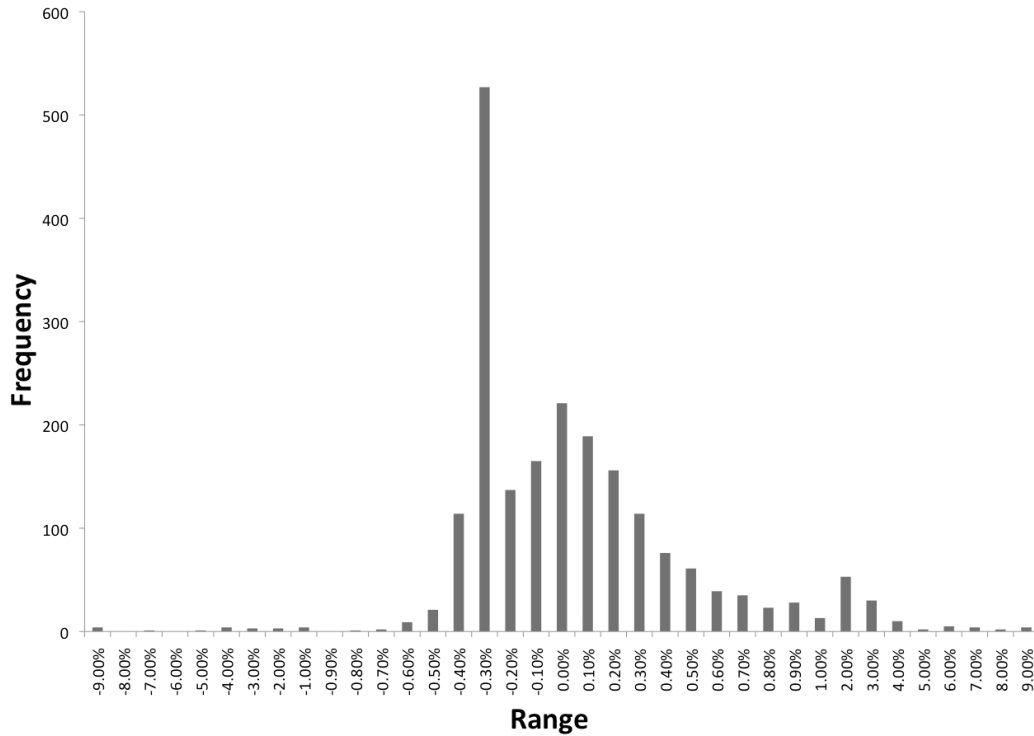


Figure 2. Histogram of Percentage Differences.

Similarly a histogram was created, as seen in Figure 3, displaying the frequencies of dollar differences between the calculations and the recorded distributions. It is seen from Figure 3 that the majority of dollar differences lay between $-\$700.00$ and $\$700.00$.

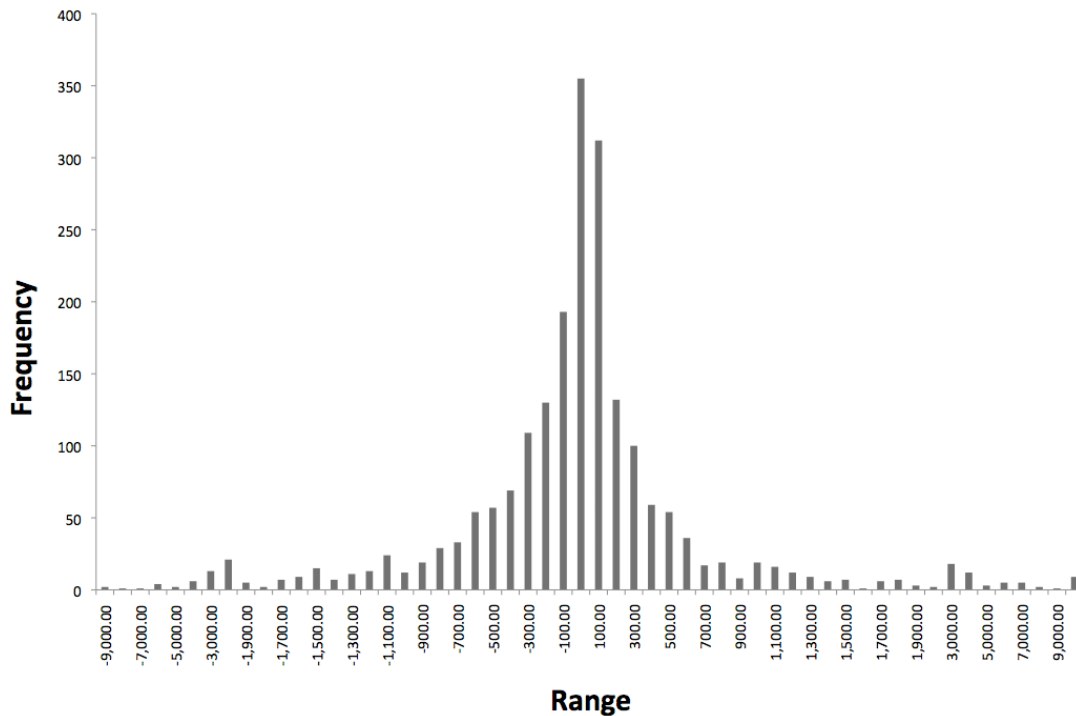


Figure 3. Histogram of Dollar Differences.

Using the two histograms above, scatter plots were constructed to display a combination of the dollar differences and percentage differences. Figure 4 displays all of the departments in the data set. The horizontal axis represents the dollar differences and the vertical axis represents the percentage differences.

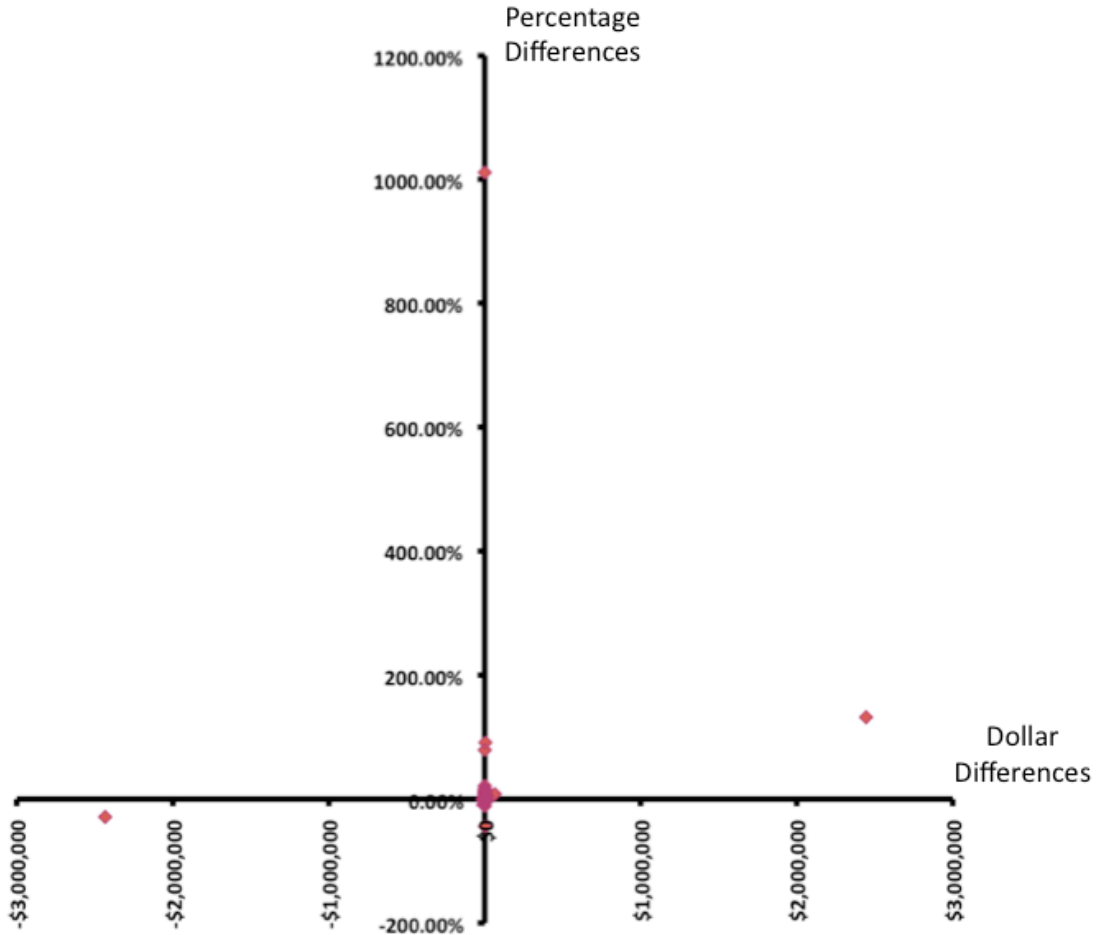


Figure 4. Plot of Dollar Differences against Percentage Differences.

As is evident from the plot, three outliers compress the rest of the data points around the origin. These outliers are the three departments that exhibited either the largest percentage differences or the largest dollar differences and are included in Tables 7 and 8 in Appendix II. In fact, these departments exhibit large differences due to manual changes implemented by RIDC after the fiscal quarter ended and are thus accounted for.

To get a better view of the compressed data, a second scatter plot was created that does not include these outliers. This plot is seen in Figure 5. Without the outliers, there is much more detail as to where the departments are positioned with regard to the exhibited dollar and percentage differences.

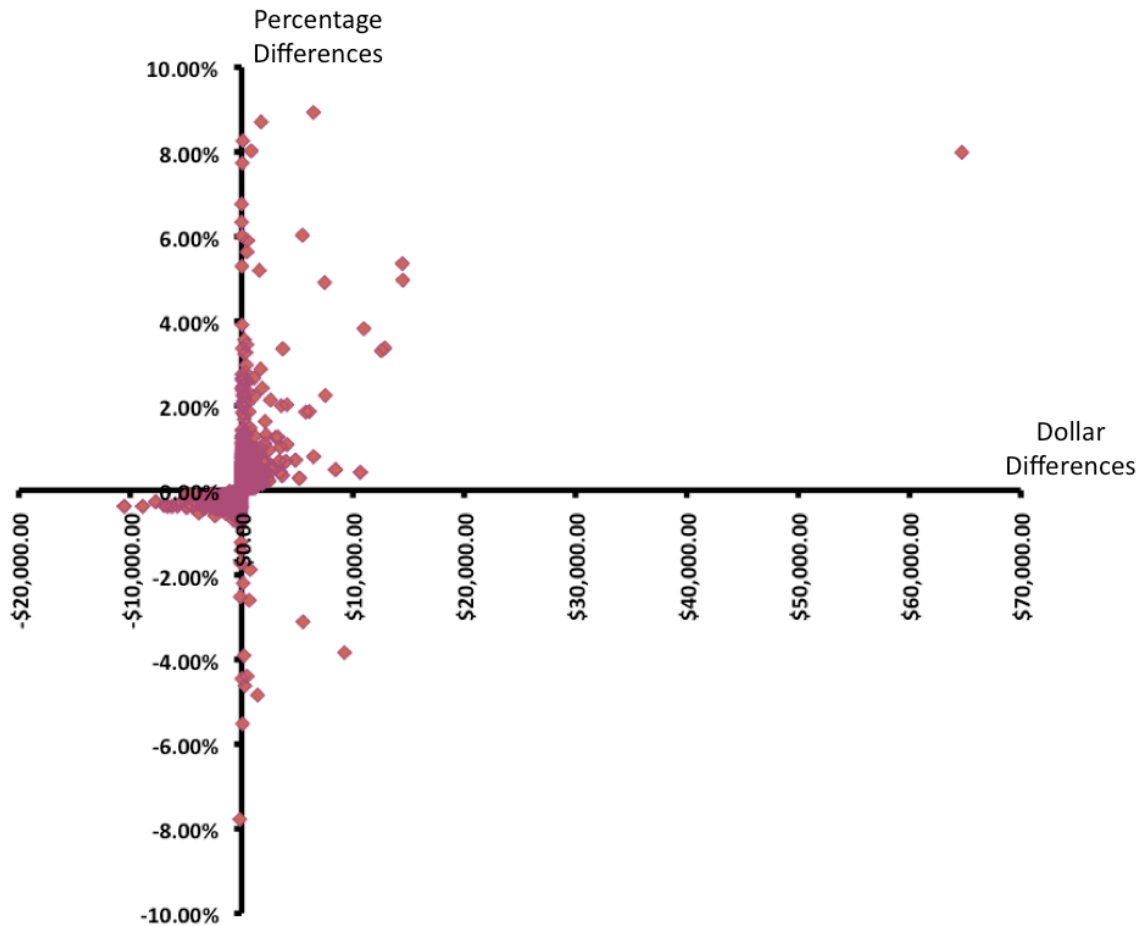


Figure 5. Enhanced Plot of Dollar Differences against Percentage Differences.

Organizing the data in these scatter plots provide a quick and easy visual displaying the dollar and percentage differences calculated in the database. In Figures 4 and 5, the data points closer to the origin correspond to smaller dollar and percentage differences. The points farther from the origin display larger differences. Confirming the results shown in Figures 2 and 3, the vast majority of the departments in the database are positioned near the origin.

Discussion

As is evident from the results, the vast majority of the departments have a recorded distribution that is similar to the calculations completed by this study. As was hoped, only a small number, namely 43 of the 2,083, just over 2.00%, of the departments exhibited percentage differences larger than 5.00% or less than -5.00%. On the other hand, 1,931 departments displayed percentage differences that ranged from -1.00% to 1.00%. It was predicted that the majority of the departments would lie in this range.

Notice that the results displayed in Figures 2 and 3, particularly in Figure 3, exhibit the form of possible normal distributions. It is not surprising to see this occur as

many outcomes and phenomena are often modeled by Gaussian normal curves [4]. Furthermore, it was expected to see the comparisons follow this type of distribution. Should the results have displayed a different type of distribution, or even no distribution, could have signaled serious discrepancies in the interest allocations.

An unexpected result was the large number of departments displaying calculated values 0.30% to 0.40% smaller than the recorded values. As mentioned in the results, it is not apparent why this occurs. Whether RIDC carries their calculations to a greater accuracy, or their procedure for calculation is slightly different than the one used in this study, the oddity is prevalent. Since RIDC requested only those departments that fell outside of the -5.00% to 5.00% range, this occurrence is more a matter of interest than it is a matter of concern.

Lastly, of those departments submitted to RIDC for investigation, all have been accounted for. Each of the 43 departments seen in Appendix II was manually adjusted by RIDC after the conclusion of the fiscal quarter. These manual adjustments arise for many different reasons. One example is a department may close their account with RIDC during the quarter, hence altering the calculation process for that department. As a result of the manual adjustments, the data recorded for the departments are skewed, thus creating the large discrepancies.

Conclusions

In closing, an analysis was conducted comparing the recorded interest dispersions and calculated values resulting from raw data provided by RIDC. Two metrics were used to display the differences. The first was the total dollar difference and the second was the percentage difference.

Using percentage difference comparisons allowed for simple identification of those departments that displayed recorded dispersions that were not within 5.00% of the calculated values in this study. It was found that only 43 of the 2,083 departments fell outside of this range.

A list of those departments was submitted to RIDC for investigation. Upon review of this list, RIDC accounted for each department. The departments sent for investigation were manually adjusted, for various reasons, after the fiscal quarter was completed. These adjustments resulted in the significant differences calculated in this study. Since RIDC has cleared each of these departments, this concludes the proceedings of this study.

Recommendations

After conducting this analysis it is recommended that RIDC execute the following:

- Perform an analysis similar to this study at the end of each quarter, particularly if there have been manual adjustments made to the departments.
- Manually calculate the values for the interest dispersions using software such as Microsoft Access™, and compare these values to the recorded distributions using percentage differences and dollar differences.
- List the departments in ascending order on a spreadsheet by percentage difference and identify those that should be investigated.
- Create histograms of the dollar and percentage differences similar to those in Figures 2 and 3. It is expected that the histograms could be modeled well by normal distribution curves. If they do not show this property, then further investigation is likely needed to identify possible inaccuracies.

Following this procedure will always identify those departments that should involve further investigation. The percentage differences are the simplest indicators of possible significant abnormalities among the databases, and as such, should be computed after every quarter.

Acknowledgements

- Anonymous This is the industrial liaison at RIDC. We would like to thank this person for proposing and allowing us to learn from this project.
- Ow, Wellington Dr. Ow is a professor emeritus of mathematics at Michigan State University and the faculty manager for this project. We thank Dr. Ow for his input, suggestions, and guidance.
- Wu, Peiru Dr. Wu is a professor of mathematics at Michigan State University and Director of the Professional Science Master's (PSM) Program in Industrial Mathematics. We thank Dr. Wu for her inputs, suggestions, and guidance throughout the report generating process.

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Appendix I: Formulas Employed by RIDC

Table 6, below, displays the formulas used to calculate the values seen in Table 3. The first three columns are extracted from the original data set provided by RIDC with the dates (d_i) converted to general numbers.

The conversion values (c_i) represent how long each account transaction is from the end of the reviewed quarter. Factors (x_i) are then derived to provide an adjusted amount for each account transaction. After calculating the factors, the original amounts for each transaction are multiplied by their corresponding factors to provide adjusted amounts (y_i). The interest subtotals (z_i) are calculated by dividing each adjusted amount by the total amount of money invested with RIDC in the reviewed quarter then multiplied by the total interest dispersed in the same quarter. Lastly the sum of the subtotals (T_k) is added up to display the interest dispersion for the k^{th} department. Table 6 displays the formulas used for Department 01010101 in the database.

Table 6. Table of Formulas for Department 01010101.

Account	Amount (\$) (a_i)	Date (d_i)	Conversion (c_i)	Factor (X_i)	Adjusted Amount (\$) (y_i)	Interest Subtotal (\$) (z_i)
Beg Bal	301,309.4	39,995	$c_1 = 40,087 - d_1$	$x_1 = c_1/92$	$y_1 = a_1 \times x_1$	$z_1 = (y_1/t) \times I$
124100	1,604.03	40,000	$c_2 = 40,087 - d_2$	$x_2 = c_2/92$	$y_2 = a_2 \times x_2$	$z_2 = (y_2/t) \times I$
401100	-269.35	40,011	$c_3 = 40,087 - d_3$	$x_3 = c_3/92$	$y_3 = a_3 \times x_3$	$z_3 = (y_3/t) \times I$
124100	1,466.09	40,014	$c_4 = 40,087 - d_4$	$x_4 = c_4/92$	$y_4 = a_4 \times x_4$	$z_4 = (y_4/t) \times I$
301100	1,466.65	40,025	$c_5 = 40,087 - d_5$	$x_5 = c_5/92$	$y_5 = a_5 \times x_5$	$z_5 = (y_5/t) \times I$
124100	-1,466.65	40,025	$c_6 = 40,087 - d_6$	$x_6 = c_6/92$	$y_6 = a_6 \times x_6$	$z_6 = (y_6/t) \times I$
401100	-269.35	40,043	$c_7 = 40,087 - d_7$	$x_7 = c_7/92$	$y_7 = a_7 \times x_7$	$z_7 = (y_7/t) \times I$
124100	1,466.65	40,043	$c_8 = 40,087 - d_8$	$x_8 = c_8/92$	$y_8 = a_8 \times x_8$	$z_8 = (y_8/t) \times I$
301100	1,467.77	40,056	$c_{10} = 40,087 - d_{10}$	$x_9 = c_9/92$	$y_9 = a_9 \times x_9$	$z_9 = (y_9/t) \times I$
124100	-1,467.77	40,056	$c_{11} = 40,087 - d_{11}$	$x_{10} = c_{10}/92$	$y_{10} = a_{10} \times x_{10}$	$z_{10} = (y_{10}/t) \times I$
124100	1,467.77	40,071	$c_{12} = 40,087 - d_{12}$	$x_{11} = c_{11}/92$	$y_{11} = a_{11} \times x_{11}$	$z_{11} = (y_{11}/t) \times I$
401100	-269.35	40,074	$c_{13} = 40,087 - d_{13}$	$x_{12} = c_{12}/92$	$y_{12} = a_{12} \times x_{12}$	$z_{12} = (y_{12}/t) \times I$
301100	1,468.33	40,086	$c_{14} = 40,087 - d_{14}$	$x_{13} = c_{13}/92$	$y_{13} = a_{13} \times x_{13}$	$z_{13} = (y_{13}/t) \times I$
124100	-1,468.33	40,086	$c_{15} = 40,087 - d_{15}$	$x_{14} = c_{14}/92$	$y_{14} = a_{14} \times x_{14}$	$z_{14} = (y_{14}/t) \times I$
Sum of Subtotals (\$)						$T_k = \sum_i z_i$
$t = \$4,691,538,822.52$		$I = \$431,887,552.18$				

Appendix II: Departments Outside of Accepted Range

Table 7. List of Departments with a Greater Than 5.00% Percentage Difference in Descending Order.

Department	MSU Allocation (\$) (T_k)	RIDC Allocation (\$)(R_k)	Difference (\$) (D_k)	Difference (%) (P_k)
11170101	40.44	3.64	36.80	1,010.99%
25210132	4,298,380.55	1,853,515.33	2,444,865.22	131.90%
33030194	8,024.35	4,198.72	3,825.63	91.11%
080101HB	40.44	22.55	17.89	79.33%
82260120	664.31	370.94	293.37	79.09%
690201HB	190.50	156.59	33.91	21.65%
10010111	135.61	115.14	20.47	17.77%
82110118	202.12	173.53	28.59	16.47%
02030120	344.62	299.00	45.62	15.26%
24060101	29,613.01	25,919.50	3,693.51	14.25%
73110131	1,551.86	1,365.09	186.77	13.68%
01010111	29,559.28	26,108.49	3,450.79	13.22%
33030195	61,323.32	54,429.31	6,894.01	12.67%
52160101	61,727.62	54,825.69	6,901.93	12.59%
82110119	3,258.14	2,919.80	338.34	11.59%
33030196	52,337.72	46,905.17	5,432.55	11.58%
690201HA	213.39	191.27	22.12	11.56%
08010112	1,149.91	1,038.83	111.08	10.69%
77020114	2,925.28	2,648.84	276.44	10.44%
080101HA	130.66	119.29	11.37	9.53%
55020117	18,567.78	16,967.75	1,600.03	9.43%
190101HA	1,283.40	1,176.02	107.38	9.13%
82510102	78,800.55	72,337.86	6,462.69	8.93%
82160115	21,934.84	20,177.79	1,757.05	8.71%
63040112	1,580.32	1,459.74	120.58	8.26%
82060211	11,699.57	10,830.53	869.04	8.02%
49020103	874,693.38	809,989.05	64,704.33	7.99%
770201HA	964.85	895.57	69.28	7.74%
730601HA	112.17	105.06	7.11	6.77%
080101HC	175.61	165.14	10.47	6.34%
82060110	96,262.57	90,789.02	5,473.55	6.03%
38030111	1,079.44	1,018.24	61.20	6.01%
27040110	9,972.42	9,416.41	556.01	5.90%
63030121	9,192.86	8,702.40	490.46	5.64%
33050110	283,987.78	269,540.65	14,447.13	5.36%
73070112	769.27	730.56	38.71	5.30%
63240120	32,416.17	30,814.28	1,601.89	5.20%

Table 8. List of Departments with a Less Than –5.00% Percentage Difference in Descending Order.

Department	MSU Allocation (\$) (T_k)	RIDC Allocation (\$) (R_k)	Difference (\$) (D_k)	Difference (%) (P_k)
81100112	-1,664.49	-1,761.75	97.26	-5.52%
20030112	1,856.06	2,012.63	-156.57	-7.78%
33050120	63,999.80	70,498.63	-6,498.83	-9.22%
20020112	-475.67	-536.53	60.86	-11.34%
25210131	6,004,691.71	8,438,195.48	-2,433,503.77	-28.84%
51030111	-1,576.76	-2,819.27	1,242.51	-44.07%