Cam Design for Stryker Medical's BIG WHEEL™

Brian R. Armstrong
Jeffery K. Barron
Adam M. Nohel

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Abstract. We constructed free-body diagrams for the cam mechanism used in engaging the BIG WHEEL™ developed by Stryker Medical. We derived a cam profile as a function of certain parameters using the constraints of the bar links, pinned joints, and ranges of motion. Once this cam profile function was derived, we designed four cam profiles and investigated the moments of each. A cam that equalized the moment (or torque) and a cam that constrained the standard deviation of incremental work to be zero were included. Finally, we computed the radius of curvature for the cam profiles we derived.

Work done for Stryker Medical under the direction of Michael Hernandez, Associate Project Manager, in partial fulfillment of the requirements of Michigan State University MTH 890 or ME 890, advised by Professor Steven Shaw, Department of Mechanical Engineering.

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Introduction

Stryker Medical manufactures, sells, and services specialty medical equipment for the treatment, transfer, and transport of patients. Stryker's line of products includes ambulance cots, stretchers, sleep surfaces, and patient room furniture [1]. One of Stryker's innovative stretcher designs includes the BIG WHEEL™ option, which increases the mobility and decreases the effort of pushing the stretcher (Figure 1).

Figure 1. The Emergency Room Transport Stretcher with BIG WHEEL™ serves in patient transport and recovery.

Our goal was to design a cam (derive a cam profile) which drives the vertical location of a roller under load so that the force input from a four bar linkage [2]. The starting position of the stretcher has the four corner "free-turning" wheels on the floor (that is, wheels that can swivel 360 degrees). The force is then initiated from a person pressing a foot-pedal. The result is the BIG WHEEL™ lowering to touch the floor (that is, for a moment all six wheels are in contact with the floor), and then the final force to lift the two corner "free-turning" wheels closest to the BIG WHEEL™ off the floor. The engaging system must be able to lift a 700-pound patient and the weight of the stretcher.

The engaging system of the BIG WHEEL™ is a four bar linkage that drives a cam. The prototype cam as designed by Chris Gentile of Stryker Medical can be seen in Appendix A (Figures 14, 15, and 16). We designed free-body diagrams to determine the torque (or moments) and related forces. We also designed cam profiles and investigated the moments of these various cams. We used geometry to show relationships to the lengths of the bars, locations of pinned or fixed points, and angles at which the cam and bar links turned.

Kinematics Analysis of the Cam Design

The first step in designing our cam profile was to derive the kinematics of the linkage, that is, the geometry and motion without considering forces [3]. We wanted to have the motion of our linkage in terms of one variable.

We set up a fixed *x* and *y* coordinate system relative to space centered at point *O* (Figure 2, see also Figure 17 in Appendix B). We assumed the roller at *R* is a point (analysis of *R* as a roller with set radius is considered later).

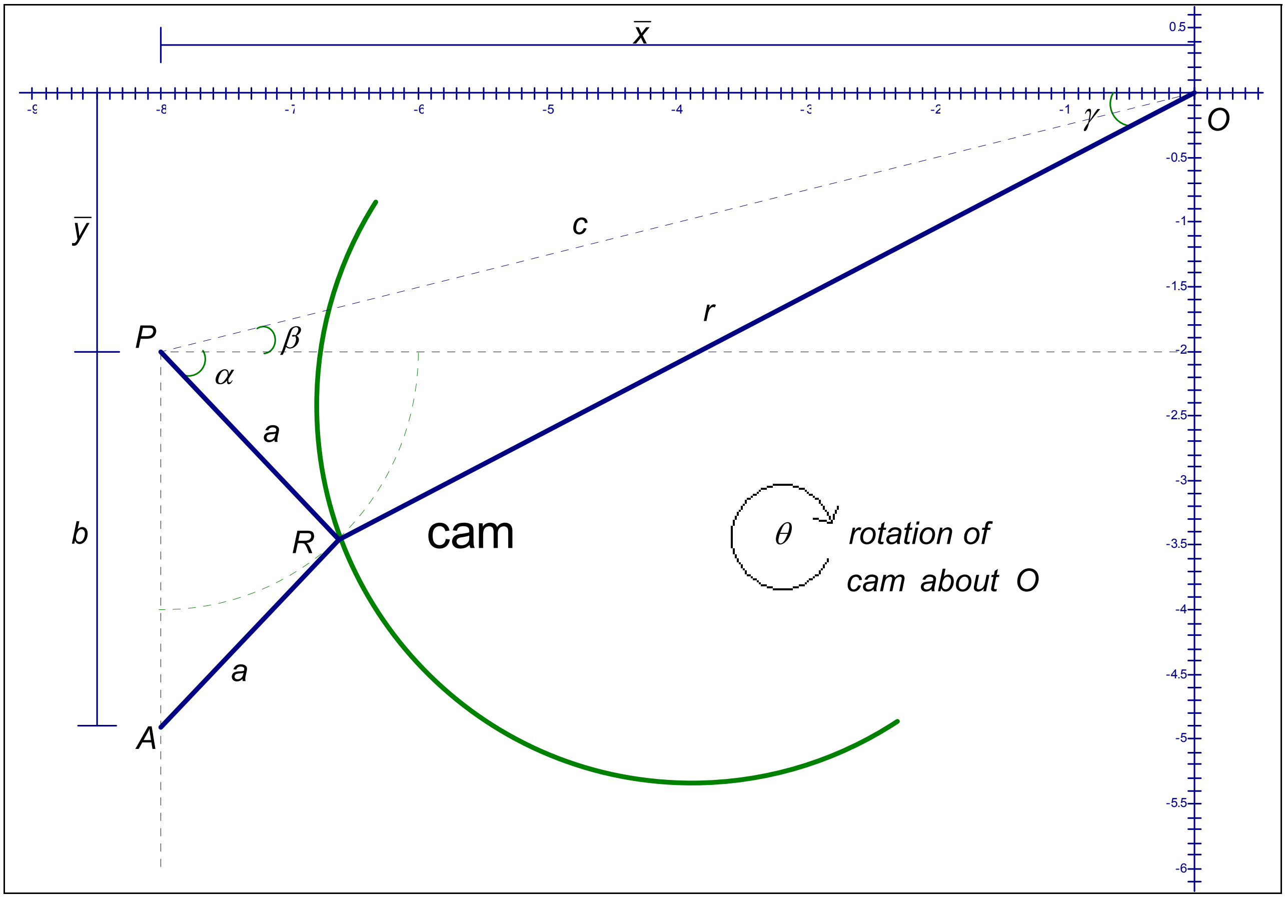

Figure 2. The diagram design shows relationships among components (produced with Geometers Sketchpad®).

The length of the segment *AP* is *b*, $b_0 = \min b \le b \le \max b = 2a$. The lengths of the segments *PO*, *PR*, and *AR* are constant throughout the movement of the cam. (Note: the segments *PR* and *AR* meet at point *R*, and are called the "scissor bars" of the system.) The lengths $\bar{x}$ and $\bar{y}$ are also constant. The angle β is fixed throughout movement. The angle γ, $\min \gamma \le \gamma \le \max \gamma$, represents the angle from the *x*-axis to the segment *OR* throughout the movement of the cam.

We considered the diagram in Figure 2 as the initial position of our mechanism. From there we derived the following relationship:

$$b(\alpha) = 2a \sin \alpha \,, \tag{1}$$

which yields $\alpha = \sin^{-1} \dfrac{b}{2a}$. Given the vertical travel of *A* to be 1.0925 inches, 0.75 inches to compress the spring, 0.3425 inches to lift the stretcher, we know upper and lower limits for b. We therefore know upper and lower limits for α. By the law of cosines,

$$r(\alpha) = \sqrt{a^2 + c^2 - 2ac\cos(\alpha + \beta)}\,, \tag{2}$$

where $r(\alpha)$ is the radius of the cam for a given angle α. The cam profile is defined with this function $r(\alpha)$, $\min r(\alpha) \le r(\alpha) \le \max r(\alpha)$, which gives the distance from the point O to the contact point R on the edge of the cam.

Now we know the lengths of the radii of the cam, but not the orientation of these radii on the cam, that is, the angle θ between consecutive radii.

Force Analysis of Cam System

Consider the free-body diagram constructed in Figure 3. We are assuming no friction and that our linkage is quasi-static.

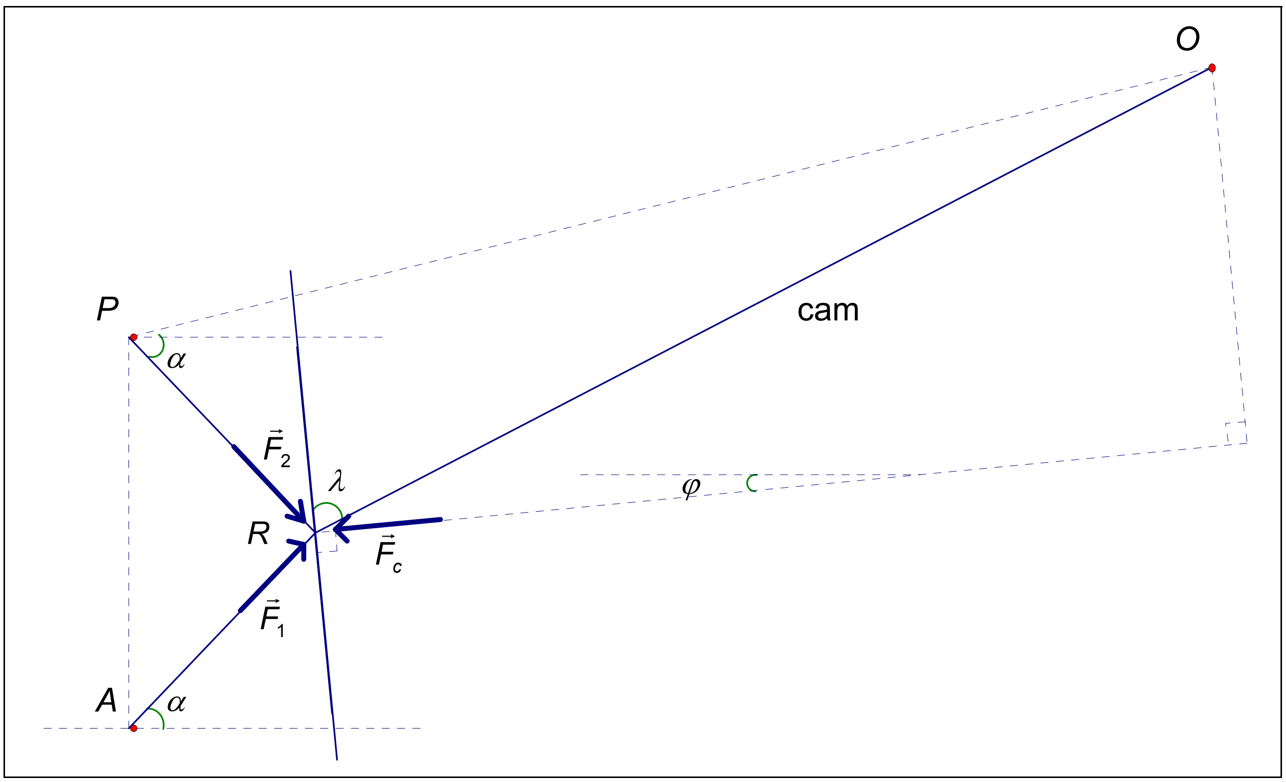

Figure 3. The free-body diagram shows the applicable force vectors and angles (produced with Geometers Sketchpad®).

The angle $\lambda = \lambda(\alpha)$ (that is, λ depends on α) measures the angle from the segment OR and the tangent line to the cam at the point of contact R. Since we are assuming no friction, the resultant force from the cam must act normal to the profile of the cam, hence the need for the angle λ. The angle φ gives the direction of $\vec{F}_c$ where $\varphi = \lambda + \gamma - 90$ degrees.

The forces shown in the free-body diagram in Figure 3 give the following relationships:

$$\begin{aligned} \vec{F}_1 &= F_1(\cos\alpha + \sin\alpha), \\ \vec{F}_2 &= F_2(\cos\alpha - \sin\alpha), \quad \text{and} \\ \vec{F}_c &= F_c(-\cos\varphi - \sin\varphi). \end{aligned} \tag{3}$$

We must have the sum of the forces in both the *x* and *y* directions equal to zero, that is,

$$\begin{aligned} \sum F_x &= F_1\cos\alpha + F_2\cos\alpha - F_c\cos\varphi = 0, \\ \sum F_y &= F_1\sin\alpha - F_2\sin\alpha - F_c\sin\varphi = 0, \end{aligned} \tag{4a,b}$$

and the moment about point *O* as a result of F_C is defined by

$$M_O \equiv F_C r(\alpha)\cos(\lambda). \tag{5}$$

Solving (4a,b) for F_C, we derived the following moment equation (M_O depending on α and λ):

$$M_O(\alpha,\lambda) = \frac{2F_1\cos(\alpha)r(\alpha)\cos(\lambda)}{\dfrac{\cos(\alpha)\sin(\lambda+\gamma-90)}{\sin(\alpha)} + \cos(\lambda+\gamma-90)}. \tag{6}$$

A First Cut at a Cam Profile

We used MS Excel as a tool for numerically computing experimental values of *r* given initial conditions and constants from the geometry of our design. The fixed values and the initial value of *b* are shown on the first line of Table 1. The range of values for *b* was $2.9075 \le b \le 4.0000$, which constrains the physical movement of the “scissor bars.”

Table 1. The initial values for the cam
from MS Excel are shown below.

$\bar{x}$	$\bar{y}$	a	c	β	b_0
-8	-2	2	8.2462	14.0362	2.9075

					coordinates	
b	α	$r(\alpha)$	γ	θ increment	x profile	y profile
2.9075	46.6251	7.4760	27.6401	1.3158	-6.6228	-3.4682
2.9575	47.6782	7.5115	27.7137	1.3158	-6.5725	-3.6366
3.0075	48.7531	7.5479	27.7829	1.3158	-6.5187	-3.8049
…	…	…	…	…	…	…
3.9900	85.9477	8.8192	27.0414	1.3158	-2.2813	-8.51890
4.000	90.0000	8.9475	26.6584	1.3158	-2.1154	-8.69384

The function $r(\alpha)$ (see equation 2) was used to numerically derive an experimental cam profile. The cam profile was constructed with constant increments of θ for each $r(\alpha)$, and the related x and y coordinates were plotted (Figure 4). We incremented θ by 1.315789 degrees for each new value. The cam must rotate a total of θ = 50 degrees, and one-to-one correspondence to the turning of the foot pedal.

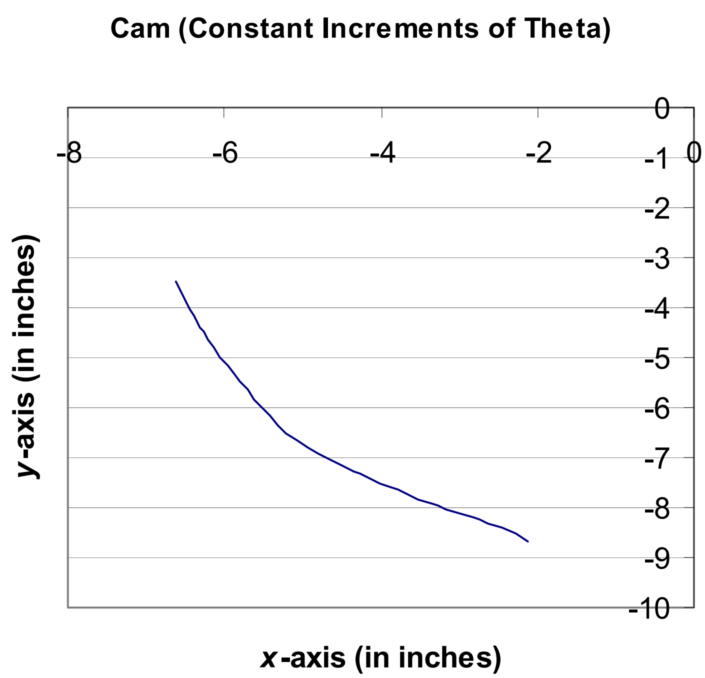

Figure 4. The numerically derived cam profile from
MS Excel with constant increasing θ is shown above.

The above cam profile in Figure 4 does not include the applicable forces. After we found the profile of the cam, we were able to consider the forces in the free-body diagram (Figure 3). There is an input force from the pedal from someone's foot that turns the cam (using the four-bar mechanism), and activates the system to engage the stretcher's BIG WHEEL™. We used our function $r(\alpha)$ and related it to the line tangent to the contact point of the cam, R, with the angle λ.

Notice the properties of the moment (Table 2) for the trial cam (Figure 4). The moment decreases from $M_O = 17.537$, where the initial value of $\alpha = 46.625$ degrees, to $M_O = 6.683$, where $\alpha = 64.406$ degrees (Table 2). There we have a jump to a moment of $M_O = 48.541$, and the moment decreases until we reach the max $\alpha = 90$, where $M_O \approx 0$.

Table 2. The Moments for given values of α and λ is shown below.

α	λ	Moment, M_o	Increasing (↑) or Decreasing (↓) Moments, M_o
46.62505	89.34211	17.53733	
47.6782	89.34211	16.48749	↓
…	…	…	…
62.79481	89.34211	7.302059	↓
64.40563	89.34211	6.683232	↓
66.11712	89.34211	48.54088	↑
67.95281	89.34211	54.49342	↑
69.94686	89.34211	57.84831	↑
72.15238	89.34211	50.04502	↓
72.74615	89.34211	48.03075	↓
…	…	…	…
84.26803	89.34211	14.26346	↓
85.94773	89.34211	9.959531	↓
90	89.34211	6.13×10^{-6}	↓

The first part of the moment where we have decreasing values represents the 0.75 inches where the cam system is depressing the spring. The jump in moment represents the value of α where the BIG WHEEL™ makes contact with the floor. Table 2 shows selected values of α where the moments are either increasing (↑) or degreasing (↓). The plot of the moment profile is shown in Figure 5.

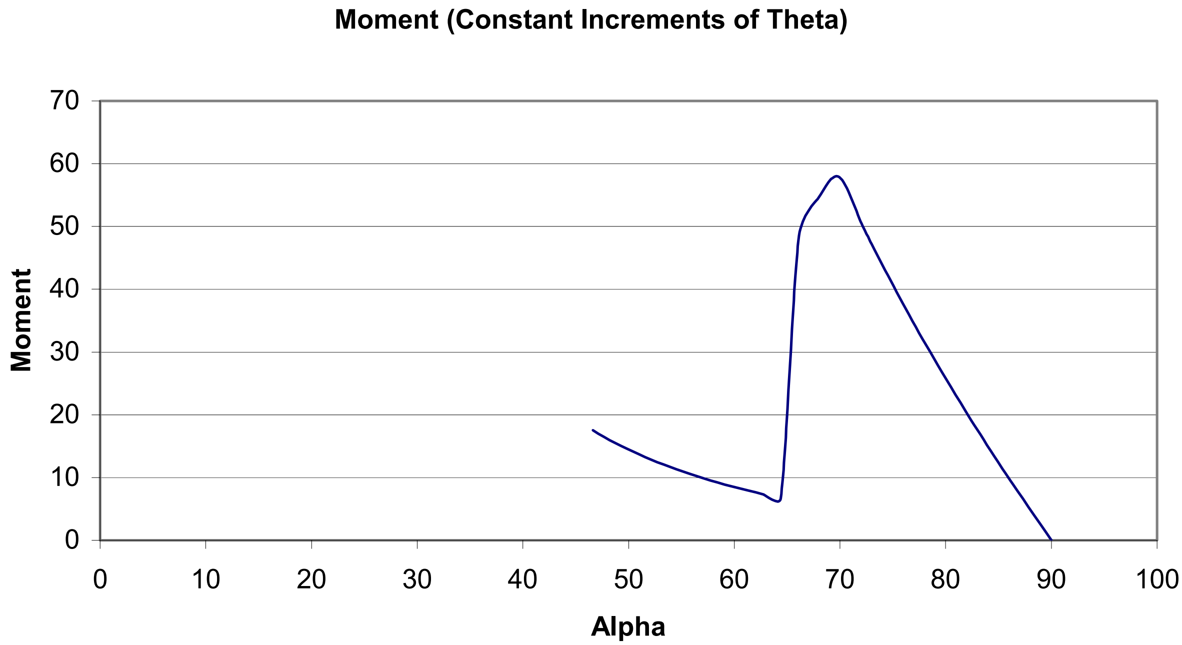

Figure 5. Moment profile for trial cam with constant increments of θ is shown above.

"Common Sense" Cam Profile

We next turn our attention to what is believed to be the "common sense" cam. Like the previous cam, there were no tools used to develop the cam profile, it was designed as another test cam, compared to those developed later. The motivation behind the common sense cam was that the cam should have little rotation against the spring, leaving most of the rotation to work against the lifting of the stretcher.

Pictured below in Figure 6 is the profile of the cam, having θ = 8 degrees of rotation devoted to compressing the spring and θ = 42 degrees of rotation for lifting the stretcher.

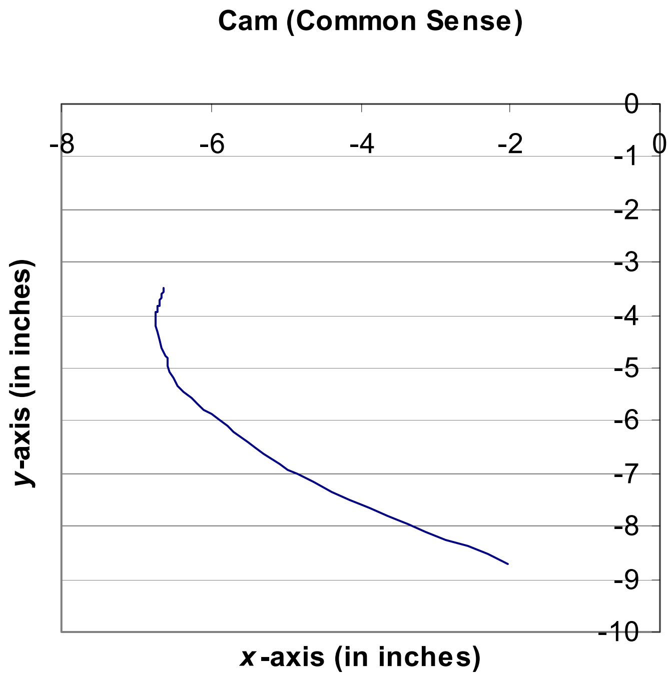

Figure 6. The numerically derived “common sense” cam profile from MS Excel is shown above.

We again wanted to consider the Moment profile for the “common sense” cam, pictured in Figure 7.

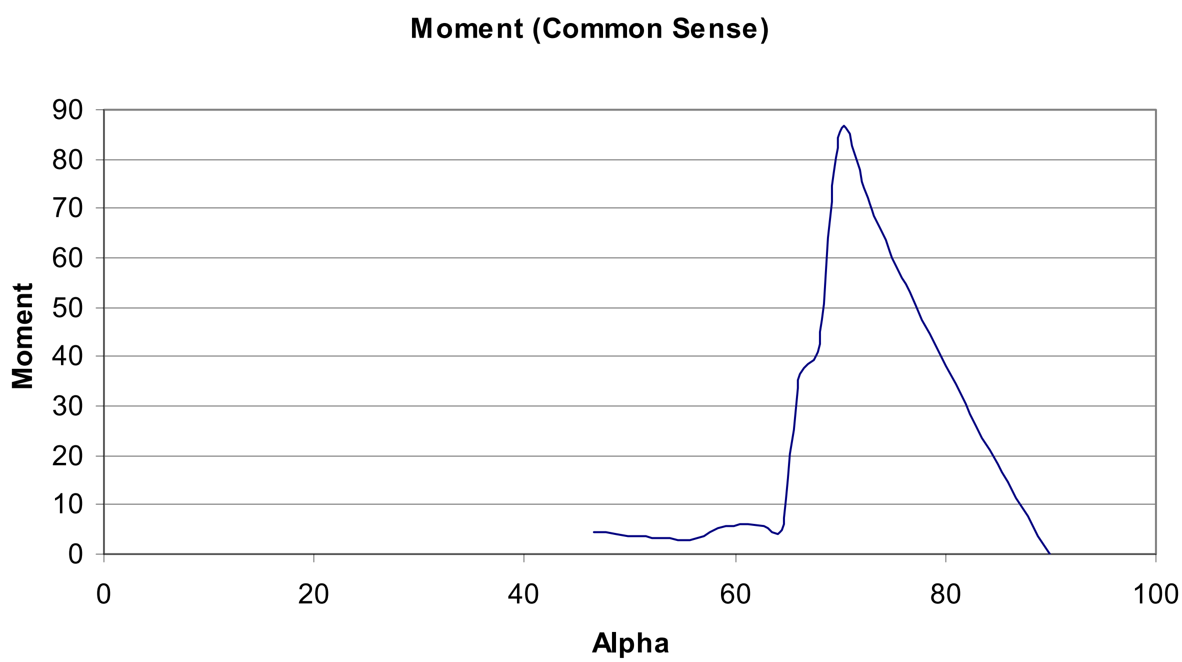

Figure 7. Moment profile for “common sense” cam with is shown above.

Constant Moment Construction of a Cam

We utilized MS Excel to find a profile that would give us a constant moment. We found the λ values that would give a constant moment, and then from the λ values we worked backward to construct the cam profile given these parameters. MS Excel allows one to perform a search over specified variables (or cells) in order to maximize, minimize, or set a value to a *target cell* that depends on those variables. We took the sum of squared differences of consecutive moments as our cost function *J*, i.e.,

$$J = \sum_i \left(M_i - M_{i-1}\right)^2 . \tag{7}$$

If the moment was constant, the summation would equal zero. This was our *target cell* in Excel. After performing the search, the cam profile in Figure 8 was produced.

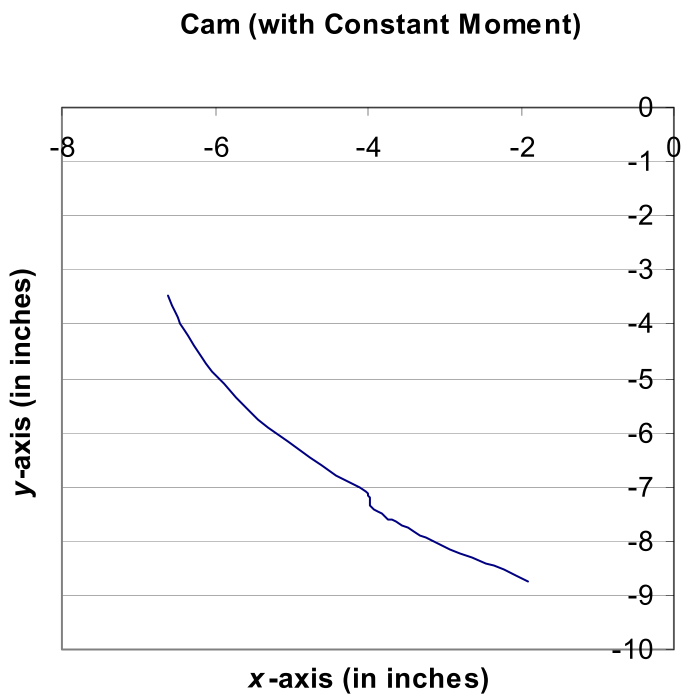

Figure 8. Cam profile with constant moment, centered at *O*, is shown above.

The "bump" in the curve represents when the BIG WHEEL™ touches the floor, that is, when the input force required changes from 50 pounds to 600 pounds (including a transition phase of 400 to 500 pounds). The accompanying moment profile can be seen in Figure 9 below.

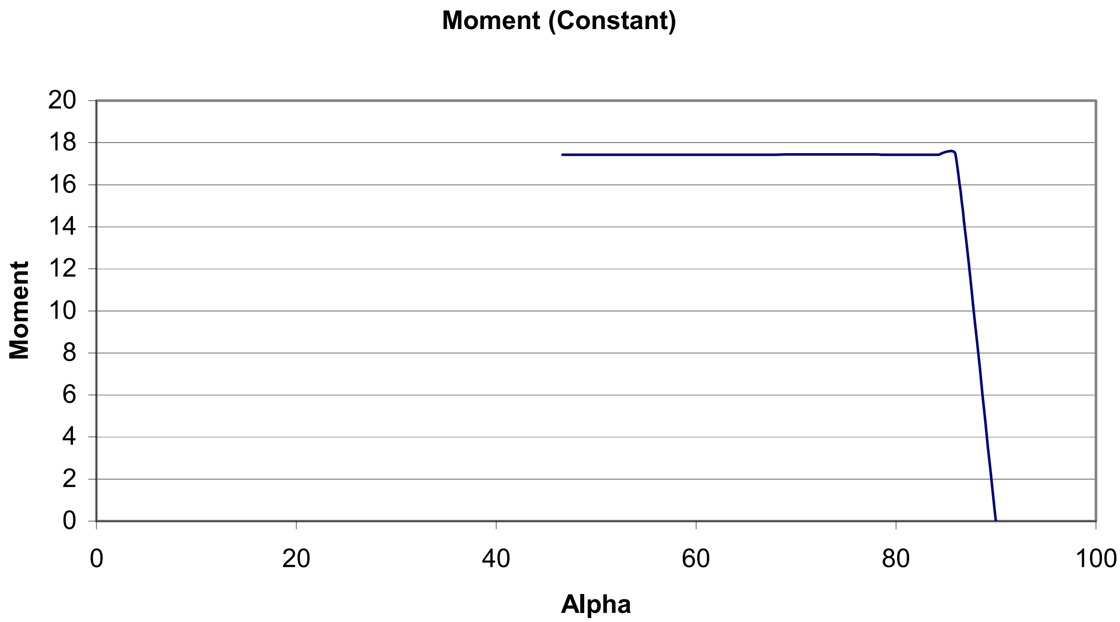

Figure 9. Constant moment profile is shown above.

The moment profile in Figure 9 shows that the moment is constant until the end where the moment falls immediately to zero. This sudden change to zero represents when the “scissor bars” are vertical. So the force from the floor is going up through the “scissor bars” and there is no longer any torque on the cam.

Cam Profile with Zero Standard Deviation of Incremental Work

We considered one more method for constructing cam profile by considering the work used to engage the cam system. In a rotational mechanical system, the work is found by the integral of torque acting through an angular displacement, that is,

$$W \equiv \int M_O(\alpha, \lambda) d\alpha \ , \tag{8}$$

where W is the total work [3]. We computed the work numerically in Excel using Riemann sums.

We used the solver in Excel to search the varying values of work to find the standard deviation of work equal to zero. The standard deviation was stored in our *target cell*.

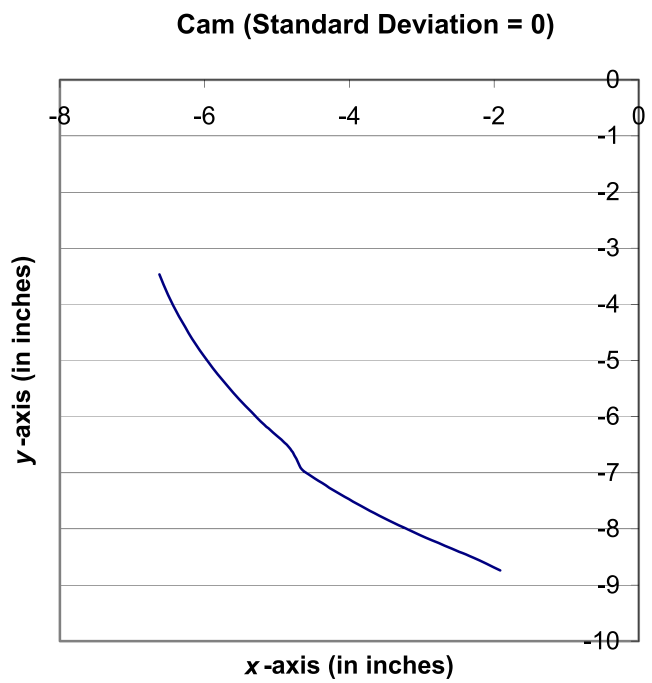

Figure 10. Cam profile with standard deviation of incremental work equal to zero is shown above.

The moment computed using the work analysis gives the following profile. The moment seems to be similar to the moment given for the constant increments of θ (Figure 5).

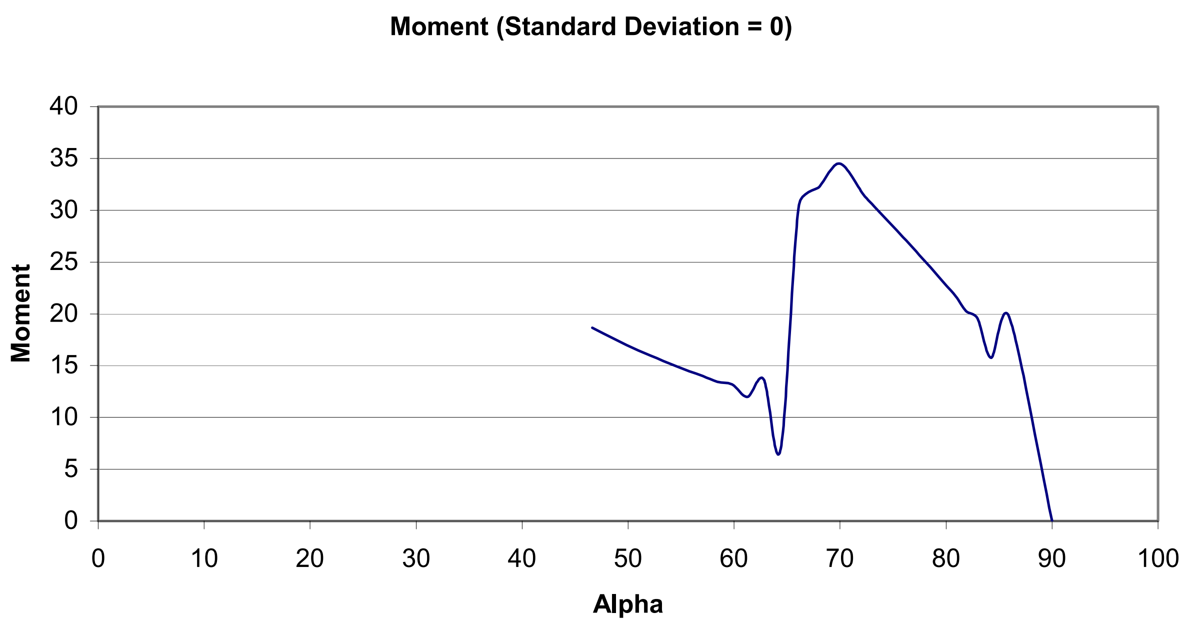

Figure 11. Moment Profile with standard deviation of work equal to zero is shown above.

Introduction of the Roller

Up to this point in our analysis, we had only looked at the situation where we considered the roller as a point at *R*. The point of contact on the cam is where the segments *PR* and *AR* meet (Figure 2).

We first specified the diameter of the roller to be 0.875 inches. We then moved the cam back 0.4375 inches (the radius of the roller) towards the origin such that the forces act normal to the tangent of the roller. We used MS Excel to simulate this analysis numerically with the discrete values of the cam profile in Figure 8. The cam profile for the roller is shown in Figure 12. The curve closest to the origin is the cam profile. The curve farthest from the origin is the path of the center point of the roller.

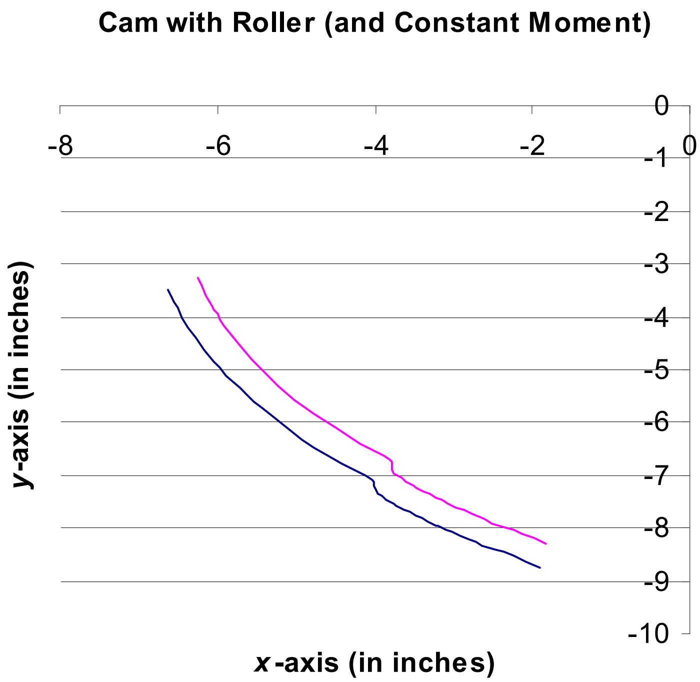

Figure 12. Cam Profile with constant moment closest to origin, and path of center point of roller farthest from origin.

For the above cam profile in Figure 12 to work, the radius of curvature must be greater than the radius of the roller. This constraint prevents binding of the system, that is, points in the curve where the roller instantaneously makes contact in two separate points on the cam.

The radius of curvature is given by

$$R_C = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|} = \frac{\left(r^2+r_\theta^2\right)^{3/2}}{\left|r^2+2r_\theta^2-rr_{\theta\theta}\right|}, \tag{9}$$

where the first part is rectangular coordinates and the second part is polar [4]. We used numerical approximations in MS Excel to compute the first and second derivatives of *y* with respect to *x*. We focused our attention to the area around the "bump" in the cam profile (Figure 12). We found the largest radius of curvature to be $R_C = 4276.761 > 0.4375$ and the smallest to be $R_C = 0.1871 < 0.4375$. However, we need $R_C > 0.4375$ for all possible values of the radius of curvature. Therefore, a roller must have a radius of less than 0.1871, but it must also be able to bear the load.

We next considered the cam profile with the roller for the zero standard deviation of incremental work. We again specified the diameter of the roller to be 0.875 inches. We moved the cam back 0.4375 inches (the radius of the roller) towards the origin such that the forces act normal to the tangent of the roller (Figure 13).

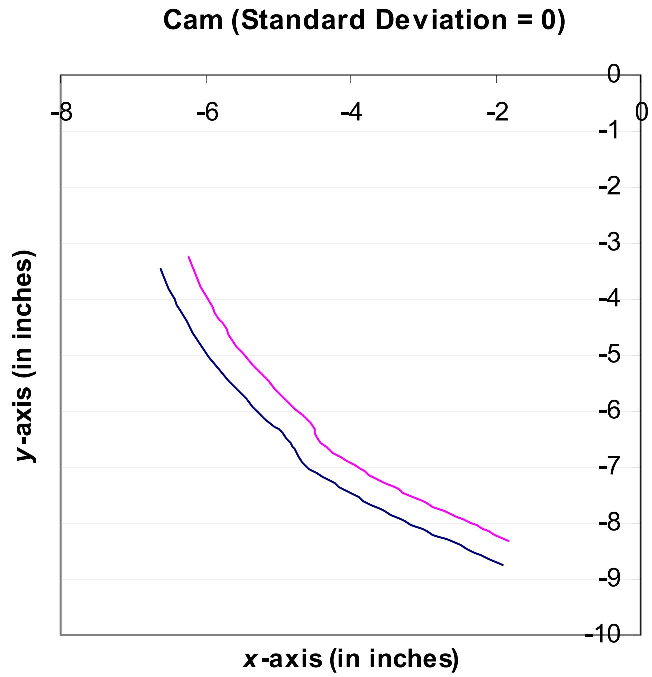

Figure 13. Cam Profile with the standard deviation of work equal to zero closest to origin, and path of center point of roller farthest from origin.

We computed the radius of curvature for the cam profile. We found the largest radius of curvature to be $R_C = 14.204 > 0.25$ and the smallest to be $R_C = 0.538 > 0.25$. The smallest radius of curvature is larger than the radius of the roller specified.

Recommendations

We recommend further testing and experimentation of the cam designs dealing with constant moment and zero standard deviation of incremental work.

Future Work

In this project, we have concentrated on cam design. The following items are additional issues, which may be addressed with the possibility of further maximizing cam design and performance.

• Suggestion One

A re-design of the foot pedal to lessen the force required. Instead of starting the turn of the pedal level, start the turning at 45 degrees above level.

• Suggestion Two

Consider the analysis including the four-bar linkage in the design of the cam. Included in the CD archive is a sample MS Excel Worksheet with the beginnings of this work.

References

[1] *Stryker Medical*. <http://www.strykermedical.com/>. 2003.

[2] Hernandez, Michael. *Project Summary*. "Cam Design (Optimization Theory)." Stryker Medical. <http://www.math.msu.edu/Graduate/msim/Project_Summaries/StrykerMedical2.htm>. 2003.

[3] Sandor, Bela Imre. *Engineering Mechanics: Statics and Dynamics*. Prentice-Hall, New Jersey. 1983.

[4] Weisstein, Eric W. "Radius of Curvature." CRC Press LLC. 1999. <http://mathworld.wolfram.com/RadiusofCurvature.html>. 2003.

Appendix A. Cam and Cam-Carriage Design (by Stryker Medical)

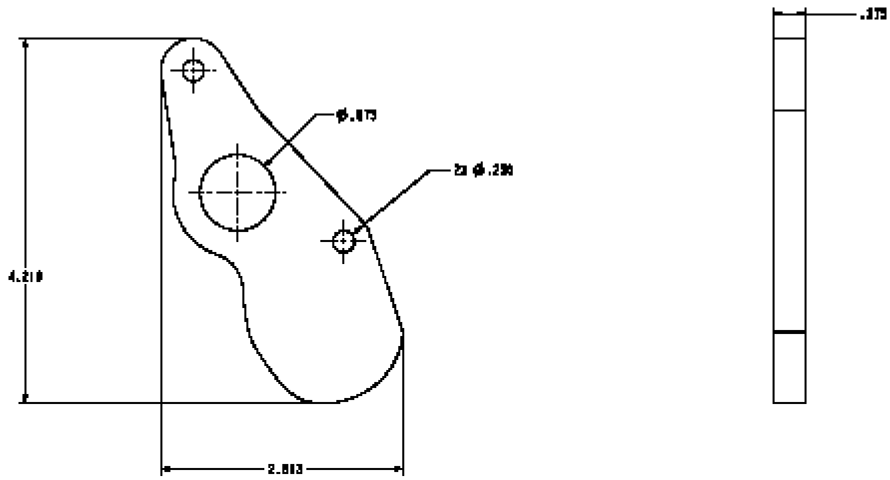

Figure 14. The cam profile as developed by Stryker Medical.

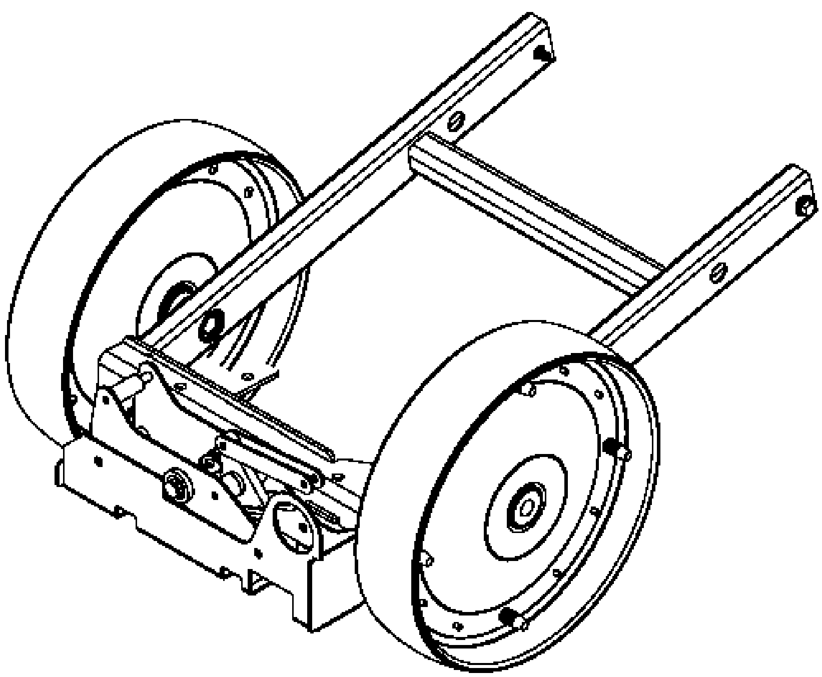

Figure 15. The cam-carriage is the engaging system of the BIG WHEEL™.

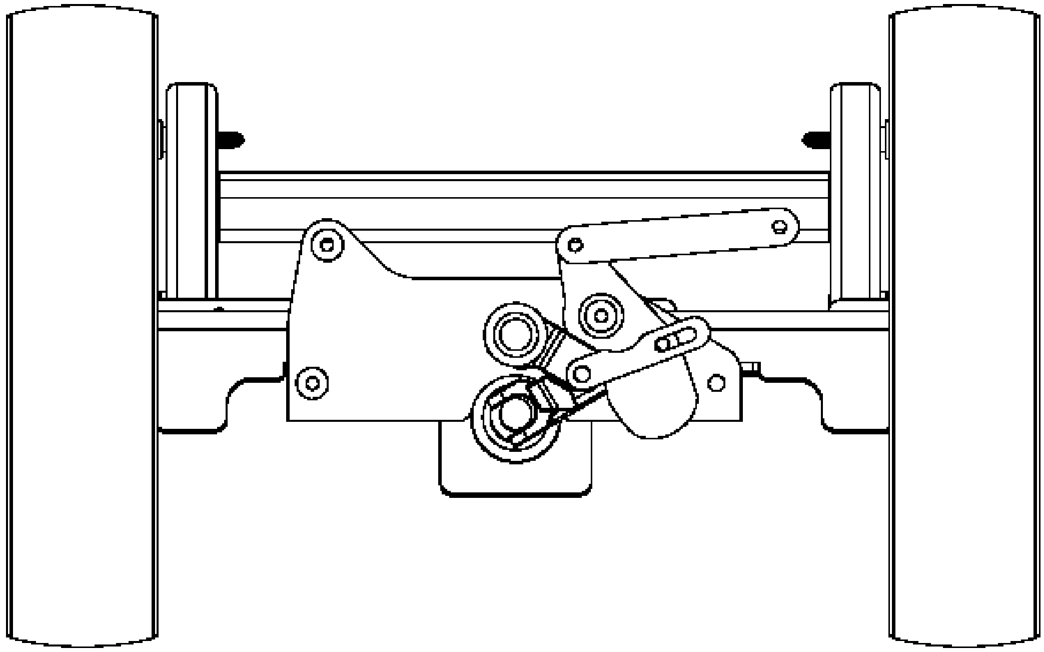

Figure 16. The cam-carriage is the engaging system of the BIG WHEEL™.

Appendix B. Experimental Cam Design Coordinate System

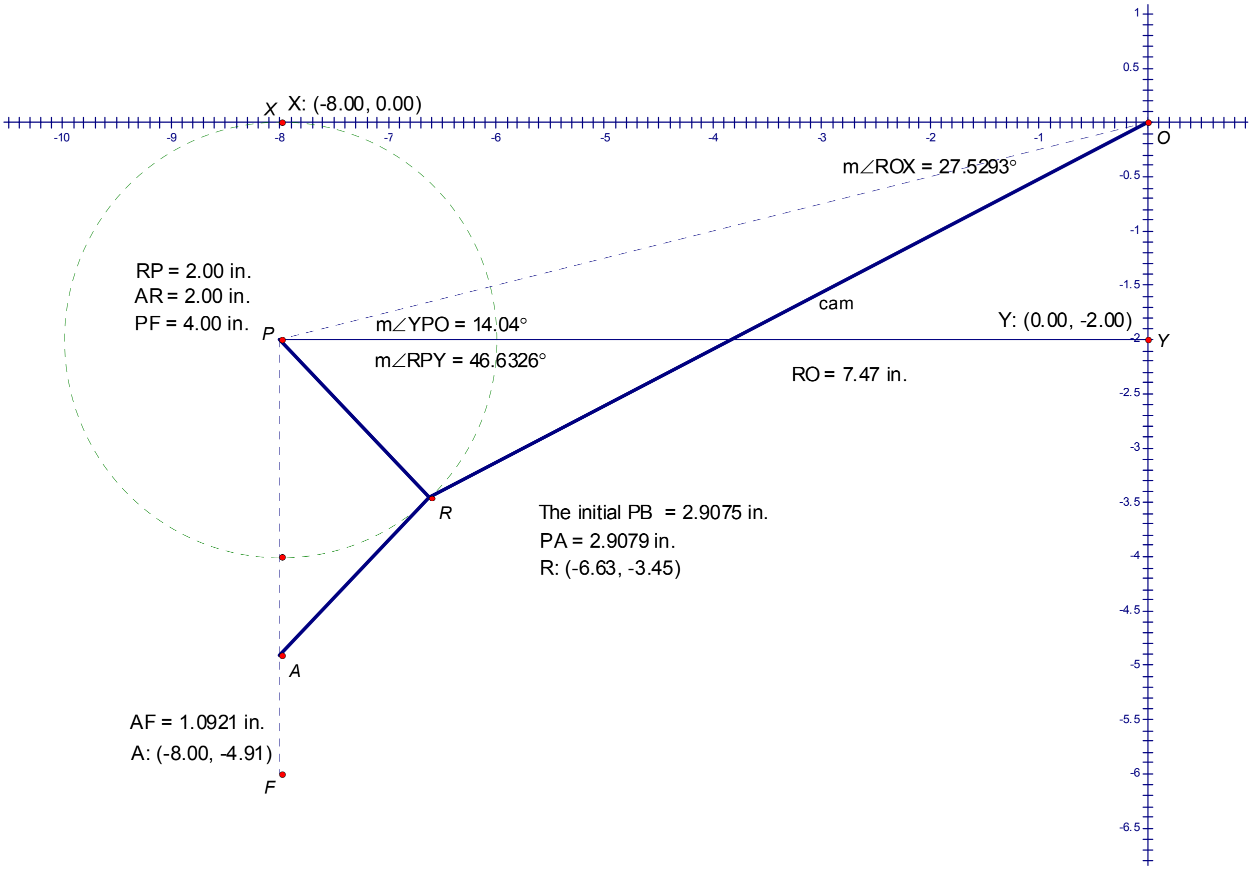

Figure 17. The cam design from Geometers Sketchpad®
shows relationships throughout movement of cam.