

Michigan State University
Department of MathematicsName: _____

PID: _____

Section No:

Signature: _____

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1. DO NOT OPEN THIS EXAM UNTIL YOU ARE INSTRUCTED TO DO SO.
2. *Without fully opening the exam*, check that you have pages 1 through 11 and that none are blank.
3. Fill in the information at the top of the page.
4. You will need a pen or pencil and this booklet for the exam. Please clear everything else from your desk.
5. The use of calculators, cell phones, or any other electronic device as an aid to writing this exam is strictly prohibited.
6. Please look to the board for possible corrections to this exam.
7. Do not spend too much time on a particular problem. Work the easier problems first.
8. The grading of this exam is based on your method. **Show all of your work.** (There are problems however that will be graded right or wrong.) If you need additional space, use the backs of the exam pages.
9. Answers can be in any form unless specified otherwise.
10. You will be given **exactly** 200 minutes for this exam.

1. (21 points) Find the derivatives of each of the following. DO NOT SIMPLIFY.

(a) $f(x) = 3^{\sec x} \tan^{-1}(5x - 1)$

(b) $g(x) = \sqrt{\cos(e^x + 1)}$

(c) $y = x^{\tan x}$

2. (12 points) Solve the initial value problem below.

$$\frac{dy}{dx} + xy = 2x, \quad y(0) = 1$$

3. (14 points) Find each of the following limits.

(a) $\lim_{x \rightarrow \infty} \left(1 + \frac{1}{3x}\right)^{5x}$

(b) $\lim_{x \rightarrow 0^+} \frac{\sqrt{x}}{\sin^{-1} \sqrt{x}}$

4. (30 points) Find each of the following integrals.

(a) $\int \frac{x^2}{\sqrt{4-x^2}} dx$

(b) $\int e^{\sqrt{x}} dx$

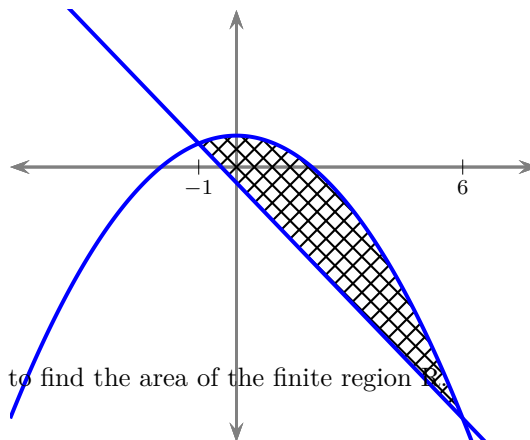
4. (con't) Find each of the following integrals.

(c) $\int \frac{x+3}{x^3-x} dx$

5. (12 points) A reservoir shaped like a right circular cone (with the point down) is 16 ft across the top and 10 ft deep. If the tank is full of water weighing 62.4 lb/ft³, how much work is required to pump the water to a point 4 ft above the top of the tank.

6. (16 points) Consider the region R bounded by curves, $y = 4 - x^2$ and $5x + y = -2$.

(a) Sketch the region R.



(b) Set up but **DO NOT EVALUATE** the definite integral to find the area of the finite region R.

(c) Set up but **DO NOT EVALUATE** the definite integral to find the volume of the solid generated by rotating the region R about the line $y = 9$.

7. (27 points) Determine whether the series below converge or diverge. *Show your work and name the tests you are using.*

(a)
$$\sum_{n=0}^{\infty} \frac{n}{\sqrt{n^2 + 1}}$$

(b)
$$\sum_{n=1}^{\infty} \frac{\ln n}{n^3}$$

7. (con't) Determine whether the series below converge or diverge. *Show your work and name the tests you are using.*

(c)
$$\sum_{n=0}^{\infty} \frac{2^n n^3}{n!}$$

8. (10 points) Find the (open) interval of convergence for the power series below. *Do not test for convergence at the endpoints.*

$$\sum_{n=1}^{\infty} \frac{x^n}{(-5)^n (n+1)}$$

9. (10 points) Find the length of the curve $y = x^{3/2}$ from $x = 0$ and $x = 1$.

10. (12 points) Find the Maclaurin series $\sum_{n=0}^{\infty} a_n x^n$ for each of the following functions. *Give an explicit formula for a_n in each part.* (You may freely use known formulas.)

(a) $\sin x^2$

(b) $(1 - x)^{-2}$

11. (12 points) Find the Taylor polynomial of order 3 generated by $f(x) = \sqrt{x+1}$ at $a = 3$.

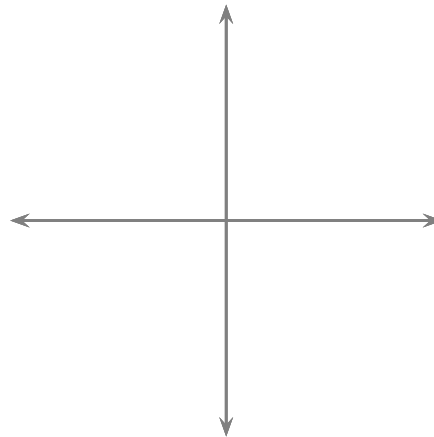
12. (8 points) Determine whether the following improper integral converges or diverges. *Justify your claim.*

$$\int_1^9 \frac{dx}{(x-1)^{2/3}} \quad \text{converges}$$

13. (16 points) Consider the functions below.

$$\begin{aligned}r_1 &= \cos \theta \\r_2 &= 1 - \cos \theta\end{aligned}$$

(a) Sketch the graphs of these functions in polar coordinates.



(b) Find all points of intersection of the graphs in (a).

(c) Set up but **DO NOT EVALUATE** the definite integral for the area which is inside r_2 and outside of r_1 .