Michigan State University
MTH 201 – Common Final Exam
Fall 2004

Student's Name ______Sample______________________________ PID__________999999______

Instructor's Name ____Keller_______________________________ Section ______All__________

DO NOT OPEN THIS TEST BOOKLET UNTIL YOU ARE TOLD TO DO SO.

NO CALCULATORS – The problems have been written with this in mind.

There are 111 possible points on this exam. A perfect paper requires only 100 points. There are nine pages including this one in the exam.

Instructions to Graders: Since there is 10% extra points on this exam and each question is worth so few points most questions are graded as right/half right/wrong. In a few cases where I have indicated you can give partial credit. If you want to stray from this rubric on a question you must let me know before continuing with the grading so that I can consider your request and its validity for all sections.

- For each problem, show enough work so that the method you used to solve the problem is clear. For questions requiring an explanation you should write a brief but complete response.
- Write all your work on these pages. If you need more room than is given with the problem, write on the back of one of the pages, and indicate where your solution can be found.
- Remember that you can ask your proctor for clarification of a question. The worst you will hear from your proctor is "sorry I can't answer that."

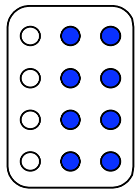
Graded by Grader #: ______________

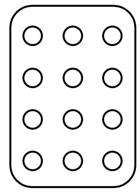
Number of points	p. 2	______	(13 possible)	p. 6	______	(14 possible)
	p. 3	______	(16 possible)	p. 7	______	(19 possible)
	p. 4	______	(12 possible)	p. 8	______	(16 possible)
	p. 5	______	(12 possible)	p. 9	______	(9 possible)

Final Total Points ______ (of 100 points)

1. In the text, there were two primary views of addition. The following two set diagrams were generated by a student as a final solution for the problem 4 + 8. **[4 points total]**

(a) Label each of the following models with the most appropriate view.





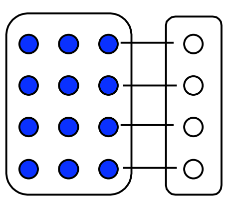
______Part whole 1 point ______joining______1 point

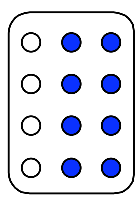
(b) Briefly describe what distinguishes these two views of addition.

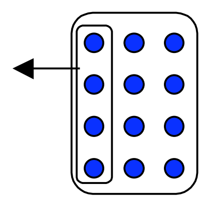
In part-whole the pieces remain distinguishable whereas in joining their origin can not be determined. [2 points – no partial credit either they have they idea or not]

2. The following three diagrams were generated by a student for the problem 12 – 4 to represent the three views of subtraction. **[5 points total]**

(a) Label each with the most appropriate view.







__1 pt each comparison ______part-whole___ take-away (separation)

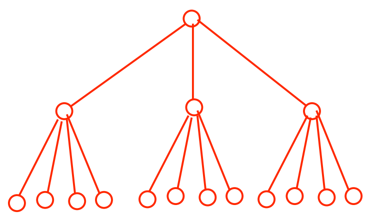
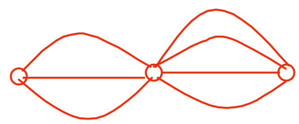
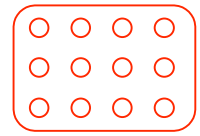
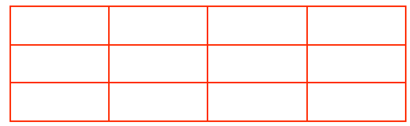
(b) For the first model only, write a story problem that corresponds to the view you have indicated.

John has 12 marbles; Mary has 4 marbles. How many more marbles does John have than Mary? Or other equivalent versions. [2 pts for a clearly worded problem suggesting comparison. Otherwise no credit]

3. Use the method of equal additions to solve the following problem. Partial credit will be given for other non-standard algorithms. **[4 points]**

$2\,{}^{1}3\,{}^{1}4$ Specifically look for adding 10 two different ways on top and on the bottom.
$-\ 1\,{}^{8}\not{7}\,9$ Give 2 pts if student is doing the correct algorithm but makes a minor subtraction
$1\ 5\ 5$ error.

4. Illustrate 3×4 using a tree, network and array model. **[6 pts total 2 pts each]**

Tree	Network	Array
		 OR 

5. **[4 points total]** Lisa calculates $63 \div 9$ as follows:

"What's $63 \div 9$? Well, 60 / 10 is 6, and 6 + 3 is 9, and 9 / 9 is 1, so its 6 + 1 = 7."

What view of division is the student using? __Sharing [2 pts]______

Explain completely the work of the student and why the answer is correct.

The student is sharing 60 among 10 groups (which is one group too many). Each group gets 6, however, still have 6 from the extra group and 3 from rounding 63 to share or 6+3 = 9. Thus each of the 9 groups gets 1 more for a total of 6 + 1 = 7. [2 pts]

6. The following problems involve converting to/from and adding in base 6.
[6 points total 2 pts right or wrong]

(a) What is the base 10 number 33 in base 6? ______53 (base 6)__________

(b) What is the base 6 number 13.25 in base 10? ______$9\frac{17}{36}$ OR $9.47\overline{2}$_____

$$1 \times 6 + 3 \times 1 + 2 \times 6^{-1} + 5 \times 6^{-2} = 9\frac{17}{36} = 9.47\overline{2}$$

(c) Using the standard algorithm for multi-digit addition but in base 6 instead of base 10, show the sum of 345 (base 6) + 123 (base 6).

$$\begin{array}{r} {}^{1}3\,{}^{1}4\,5 \\ +1\,2\,3 \\ \hline 5\,1\,2 \end{array}$$

If they do

$$\begin{array}{r} 3\,4\,5 \\ +\,1\,2\,3 \\ \hline 1\,2 \\ 1\,0\, \\ 4\,\, \\ \hline 5\,1\,2 \end{array}$$

[Gets 1 pt]

7. **[6 points total 2 pts each]** For the problem 125×840:

(a) Suppose you may round only one of the numbers to the nearest hundred to mentally ***approximate*** the answer. Which one would you choose 125 or 840 to get closer to the exact answer? Explain your thinking.

If you round 125 to 100, you are off by $25 \times 840 = 100 \times 840/4 = 21000$, whereas if you round 840 to 800, you are off by $125 \times 40 = 5000$. So round 840. To get credit, they must indicate a reason why they know $25 \times 840 > 125 \times 40$ or similar approximation. If you see that the student has computed 125×840, 100×840 and/or 125×800, then be very critical of their explanation. 2 pts for correct, 1 pts for not giving a reason for $25 \times 840 > 125 \times 40$, 1 pt for other work that shows approximation skills deserving merit.

(b) Show how number sense and mental computation can be used to get the ***exact*** answer. No credit will be given for doing all the partial products of the standard algorithm.

$125 \times 840 = 100 \times 840 + 25 \times 840 = 84000 + 100 \times 840/4 = 84000 + 21000 = 105000$
Better wording would be to do $100 \times 840 = 84000$ and add one fourth of that or 21000.
Other algorithm could be to do $8 \times 125 = 1000$, take have of this 500, then to get the answer $125 \times (800 + 40) = 100000 + 5000 = 105000$. Most other algorithms will not be worth any credit – but use your judgment. Remember it is supposed to be done mentally.

(c) Show how mental computation can be used to compute 65% of 240. Do not use decimals.

50% of 240 is 120 (taking half). 10% of 240 is 24, and 5% of 240 is half of 24 or 12. Thus, the answer is $120 + 24 + 12 = 156$. Use of benchmarks and related numbers is essential.

8. The following picture represents $\frac{3}{4}$ of an amount. How many marks would there be in $\frac{7}{6}$ of the same amount? Explain how you got your answer. **[3 points]**

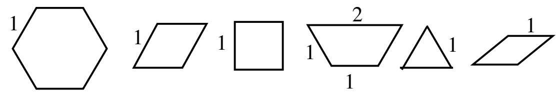
```
              *    *         *             *    *         *
         *          *   *  *       *      *   *         *  *
    *  *  *      *     *  *     *    *  *  *  *   *
```

You need to group the stars into 3 groups is this represents 3 of 4 groups (9 per group or 27 total). Adding a fourth group there would be 36. If you break 36 into 6 groups each group would have 6. To have 7 groups you would end up with 42. Note 2 pts if they do grouping but make some error. 1 pt if they do some other relevant but incorrect computation without grouping. 1 pt for $\frac{7}{6} \times \frac{4}{3} \times 27 = 42$ or similar calculation.

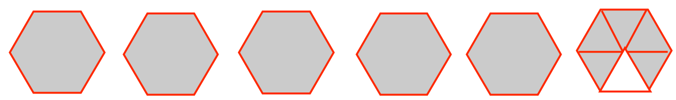
9. Which is the greater value: $\frac{9}{16}$ of a cup, or $\frac{15}{28}$ of a cup? Explain, showing your grasp of fractions and benchmarks, and without using decimals or complicated common denominators. **[3 points]**

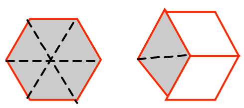
$\frac{9}{16}$ is $\frac{1}{16}$ more than $\frac{8}{16} = \frac{1}{2}$ whereas $\frac{15}{28}$ is $\frac{1}{28}$ more than $\frac{14}{28} = \frac{1}{2}$. Since $\frac{1}{16} > \frac{1}{28}$ [sharing among 16 is better than sharing among 28 to get more], $\frac{9}{16}$ is greater than $\frac{15}{28}$ or that $\frac{9}{16} > \frac{15}{28}$. Essentially right or wrong, 2 pts if they just rewrite something incorrectly but explanation is correct. 1 pt for a correct answer but shows no fraction reasoning.

10. Pattern blocks are shapes with unit sides (except for trapezoid) as shown:

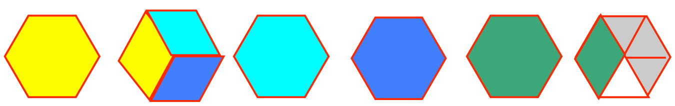


For the problem: $5\frac{5}{6} \div 1\frac{1}{3}$, illustrate the solution of this problem using pattern blocks. Explain your solution completely and indicate the whole at each stage. **[4 points]**

Using a whole of a hexagon, $5\frac{5}{6}$ is 

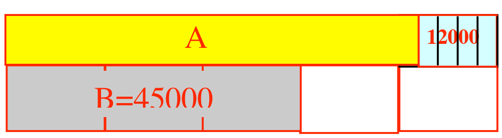
And $1\frac{1}{3}$ is  . Thus there are 4 whole $1\frac{1}{3}$'s and $\frac{3}{8}$ of whole $1\frac{1}{3}$.

So the answer is $4\frac{3}{8}$ of $1\frac{1}{3}$ in $5\frac{5}{6}$.



Note: Students must use pattern blocks otherwise no credit. Must indicate the starting whole and the whole that the answer is being interpreted. A less clear use of pattern blocks but essentially otherwise correct should get 2 pts.

11. If town A had 12,000 more people than it does, its population would be $1\frac{2}{3}$ times as big as town B is now. Town B has 45,000 people currently. What is the current population of town A? **[4 points]**



Each  has to be worth 15,000. 12,000 is 4/5 of 15,000. So A must be 4×15,000+1/5×15000 = 63,000

Or $1\frac{2}{3}$ of B would be 75,000 and that has to be 12,000 more than A. Thus A has to be 63,000. Either solution is fine. Give 2 pts for a good reasonable start that might eventually lead to a solution and otherwise 0. 2 pts for a totally algebraic solution, 4 pts if totally algebraic solution but an exceptional explanation of its relation to fractions.

12. In Boogleville, two-fifths of the men are married to one-third of the women. What is the ratio of men to women? **[4 points]**

For every married male there must be 2 unmarried women. Starting with a column for the men and adding the necessary rows for the women gives the table to the right with 2/5 of males married to 1/3 of the women. Thus the ratio of men to women is 5 to 6. Give 2 pts for a good reasonable start that might eventually lead to a solution and otherwise 0. 2 pts for a totally algebraic solution, 4 pts if totally algebraic solution but an exceptional explanation of its relation to fractions.

m/w	**w**	**w**
m/w	**w**	**w**
m		
m		
m		

13. In our work with fractions we examined decimal representations of fractions and when a decimal representation will terminate and when it will repeat. **[6 points total – 2 pts each]**

(a) Convert the repeating decimal 5.062626262626… to a fraction. $\frac{5012}{990}$

100 × 5.062626262626… = 506.26262626…
1 × 5.062626262626… = 5.062626262626
99 × 5.062626262626… = 501.2

So $99 \times 5.0\overline{62} = 501.2$ or $990 \times 5.0\overline{62} = 5012$

1 pt if minor subtraction error

(b) Give an example of a decimal representation of a number that neither terminates nor repeats. What kind of number is this?

0.010110111011110111111… or any other clearly non-terminating non-repeating decimal. [1 pt]
Irrational [1 pt] NO points for square roots of primes or π, *e*, etc without proof that they do not terminate nor repeat (unlikely). The goal of this question is to see if students can write down a "real" number that students can see does not terminate nor repeat and so irrationals must exist.

(c) Suppose that you have a three-digit number "abc". Under what conditions on a + b + c will the fraction $\frac{abc}{15}$ terminate?

To terminate in simplified form the denominator can only be products of 2 and 5. Therefore we must 'cancel' the 3 in the denominator thus the numerator must be divisible by 3. [1 pt]
Thus a + b + c must be divisible by 3. [1 pt]

14. For the numbers $m = 3^4 \times 7^2 \times 11^2 \times 13 \times 19^6$ and $n = 2 \times 3^3 \times 5^3 \times 13^4 \times 19^9$:
[8 pts total – 2 pts unless really minor error then 1 pt]

(a) Find the least common multiple of *m* and *n*. $2 \times 3^4 \times 5^3 \times 7^2 \times 11^2 \times 13^4 \times 19^9$

(b) Find the greatest common factor of *m* and *n*. $3^3 \times 13 \times 19^6$

(c) Find the number of factors of *m*. 5 × 3 × 3 × 2 × 7 = 630

(d) What is the smallest number that is a multiple of 15, 24, 33 and 60? Your answer may be expressed as a single number or as a product of numbers.

3 × 5, 2 × 2 × 2 × 3, 3 × 11, 2 × 2 × 3 × 5 2 × 2 × 2 × 3 × 5 × 11 = 1320

15. Recall that the set of whole numbers is {0, 1, 2, 3, …}. Consider the set of **composite whole numbers** (i.e. having more than 2 factors). Answer the following questions providing a justification in each case. **[14 points total – 2 pts each]** NOTE: 0 and 1 are not composite nor prime and thus not in the set. This questions tests students knowledge of this fact.

(a) Is the set of composite whole numbers closed under multiplication?

Yes, if m is a composite number $a \times b$ and n is a composite number $c \times d$, then $m \times n = a \times b \times c \times d$ which is composite OR the product of any two numbers larger than 0 and 1 will be a composite number. Composite numbers are whole numbers greater than 1 with more than 2 factors.

(b) Is the set of composite whole numbers closed under addition?

No, 4 + 9 = 13 which is prime [other examples are okay such as 8 + 9 = 17]

(c) Does an additive identity exist for the set of composite whole numbers?

No, 0 is neither prime nor composite and 0 would have to be the additive identity.

(d) Do additive inverses exist for the set of composite whole numbers?

No, can't have additive inverses without additive identity…. Give 1 pt for the statement that there are no negative numbers and (c) was correct or 2 pts if (c) was not correct and was given 0.

(e) Does a multiplicative identity exist for the set of composite whole numbers?

No, 1 is neither prime nor composite and 0 would have to be the multiplicative identity.

(f) Do multiplicative inverses exist for the set of composite whole numbers?

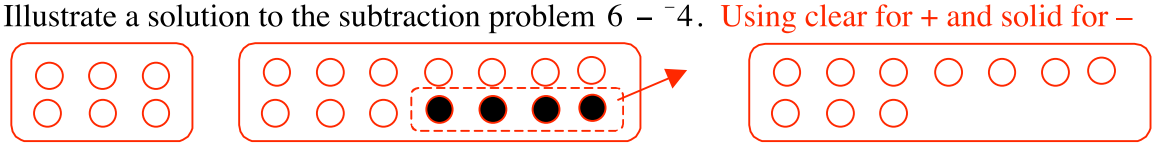
No, can't have multiplicative inverses multiplicative additive identity. Give 1 pt for the statement that there are no fractions and (e) was correct or 2 pts if (e) was not correct and was 0 credit given.

(g) What is the smallest set of numbers containing the composite whole numbers so that the answers to the corresponding questions for (a) – (f) are all true or yes? No justification is required.

Rational Numbers

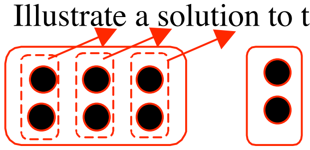
16. Using a colored chips model: **[5 points 2 pts for diagram(s) and 1 pt for view]**

(a) Illustrate a solution to the subtraction problem $6 - {}^{-}4$. Using clear for + and solid for –



6 | Also 6 but now $^{-}4$ can be removed | Yielding 10

(b) Illustrate a solution to the division problem $^{-}6 \div {}^{-}2$. What view of division are you using?



$^{-}6$ $^{-}2$

$^{-}2$ can be repeated remove 3 times. Repeated subtraction.
So $^{-}6 \div {}^{-}2 = 3$

17. Two number systems are the Roman numeration system and the Hindu-Arabic numeration system. What is a feature or trait of the indicated number system that the system has but the other system does not posses? **[4 points – 2 pts each]**

(a) Roman numeration system: has subtractive property (example IV = 4). The Hindu-Arabic system does not have this property. [Give 1 pt for multiplicative property since Hindu-Arabic has exponents and scientific notation. Both systems are additive so this gets no credit.

(b) Hindu-Arabic numeration system: has place value. The Roman numeration system has no place value. OR Hindu-Arabic has a zero, the Roman numeration system does not.

18. For the following problems involving exponents, illustrate how a student can reason the solution to the problem using only the definition of exponents for whole numbers as repeated multiplication. Then state formally in symbolic form (i.e. using *a*, *b*, *c*) the properties of arithmetic or the properties of exponents that each problem uses.
[12 points total – each square 1 pt – using a specific base in $a^b \times a^c = a^{b+c}$ can be given 1 pt**]**

Give Answer	Illustrate Solution	State Property or Properties
(a) $10^5 \times 10^4 = 10^9$	(10×10×10×10×10) ×(10×10×10×10)= 10×10×10×10×10×10×10×10×10 = 10^9	$a^b \times a^c = a^{b+c}$
(b) $\frac{10^5}{10^4} = 10^1$	$\frac{10 \times 10 \times 10 \times 10 \times 10}{10 \times 10 \times 10 \times 10} = 10$	$\frac{a^b}{a^c} = a^{b-c}$
(c) $\left(10^5\right)^4 = 10^{20}$	(10×10×10×10×10) × (10×10×10×10×10) × (10×10×10×10×10) × (10×10×10×10×10) $=10^{20}$	$\left(a^b\right)^c = a^{b\times c}$
(d) $\frac{10^5 - 10^3}{10^2} = 10^3 - 10^1$	$\frac{10 \times 10 \times 10 \times 10 \times 10 - 10 \times 10 \times 10}{10 \times 10} = \frac{10 \times 10 \times (10 \times 10 \times 10 - 10)}{10 \times 10}$ $= 10^3 - 10^1$	Distributive property of division over subtraction and $\frac{a^b}{a^c} = a^{b-c}$

19. The problems in the previous questions involve only whole number exponents. The rules stated in problem 18 can be used to motivate exponents for other types of numbers. **[9 points – 3 pts each]**

For these problems, give 3 pts for correct responses, 2 pts if the response lacks some element of rigor, 1 pt if the explanation has some element you feel is worthy of credit. However, do not hesitate to give no credit for answers that do not draw upon the work above or is in a form appropriate for elementary students.

(a) Give a rationale appropriate for elementary students for the definition for a power that is a negative integer. You may use an illustrative example such as the case of 10^{-1}.

Any use of the existing properties established above that can generate a power of –1 using only positive integers powers of 10 can be used. Here is one example

$$\frac{1}{10^1} = \frac{10^0}{10^1} = 10^{0-1} = 10^{-1}$$

Another acceptable approach is to use a pattern – for example:

$10^2 = 100,\ 10^1 = 10,\ 10^0 = 1,\ 10^{-1} = \frac{1}{10}$ This must include a discussion that on one side the exponent decreases by one and on the right side we divide by 10 each time.

(b) Give a rationale for the meaning of a number raised to a fraction using $2^{1/2}$ as an illustrative example.

If we want the property $a^b \times a^c = a^{b+c}$ to continue to hold for rational numbers, then the following would have to be true: $2^{1/2}2^{1/2}=2^1$. Thus $2^{1/2}$ can be defined as the number that multiplied by itself gives 2. Other properties can be used, but they must generate a situation that can be interpreted using positive integers.

(c) Give a justification of why $2^{1/2} = \sqrt{2}$ is irrational. [Hint: If $2^{1/2}$ is a rational number, then it can be expressed as a fraction $\frac{a}{b}$ and then $\frac{a^2}{b^2} = 2$ or $a^2 = 2b^2$. Think prime factorizations.]

One possible reasoning goes like this starting from $a^2 = 2b^2$. Consider the prime factorizations of both sides of the equation. If 2 is present on the left side then it must occur to an even power since perfect squares have prime factorizations with all even powers. This argument then gives that the left side must have an odd power for 2. A contradiction.

Of course the proof where you assume $\frac{a}{b}$ is in simplified form and then show that it can't be is okay as well.