Michigan State University
Department of Mathematics

Name: ________________________ **PID:** _________________ **Section No:** _______

Signature: ________________________

Page	Maximum Points	Actual Points
2	24	
3	34	
4	28	
5	28	
6	24	
7	16	
8	18	
9	28	
Total	200	

1. Do not open this exam until you are instructed to do so.
2. *Without fully opening the exam,* check that you have pages 1 through 9 and that none are blank.
3. Fill in the information at the top of the page.
4. Please look to the board for possible corrections to this exam.
5. Do not spend too much time on a particular problem. Work the easier problems first.
6. The grading of this exam is based on your method. **Show all of your work.** If you need additional space, use the backs of the exam pages.
7. Place your answers in the boxes, where provided. Answers can be in any form unless specified otherwise.
8. You will be given **exactly** 120 minutes for this exam.

1. (24 points) Compute each of the following limits or show that they do not exist. **Show your work.**

   (a) $\lim_{x \to -3} \frac{x+3}{x^2+7x+12}$

   (b) $\lim_{x \to 9} \frac{\sqrt{x}-3}{x-9}$

   (c) $\lim_{x \to 0} \frac{x}{\sin 5x}$

   (d) $\lim_{x \to 2^+} \frac{|x-2|}{2-x}$

2. (24 points) Find the derivative of each of the following functions. **Do not simplify.**

   (a) $f(x) = x^2\sqrt{\sin x}$

   (b) $g(x) = \dfrac{x^2 + x + 1}{\sqrt{x^2 + 1}}$

   (c) $p(x) = \displaystyle\int_1^{\sqrt{x}} \sin^5 t\, dt$

3. (10 points) Let $f(x) = \dfrac{1}{3x}$. Use the *definition of the derivative* to compute $f'(2)$. **Show your work.**

4. (14 points) Evaluate each of the following indefinite integrals. **Show your work.**

(a) $\int x^2\sqrt{1+10x^3}\,dx$

(b) $\int \frac{x}{\sqrt{x+1}}\,dx$

5. (14 points) Evaluate each of the following definite integrals. **Show your work.**

(a) $\int_0^1 (x^2+1)(3x-2)\,dx$

(b) $\int_0^{\pi} \sin x\, \cos^2 x\,dx$

6. (12 points) Find the equation of the tangent line to the curve $x^3 + y^3 = 9xy$ at the point (4, 2). **Show your work.**

7. (16 points) 100 m^3 of oil is spilled when a tanker collides with a tuna boat. The resulting oil slick forms a right circular cylinder on the surface of the water. If the thickness (h) of the slick is *decreasing* at a rate of 0.001 m/sec, how fast is the radius (r) increasing when the slick is 0.01 m thick? *Note:* $V = \pi r^2 h$

8. (16 points) A rectangular box with volume 18 ft^3 is to be built with a **square** base and *NO* top. The material used for the bottom panel costs \$2 per ft^2 while the material used for the side panels costs \$1.50 per ft^2. Find the minimum cost of such a box. **Justify your answer using the methods of calculus.**

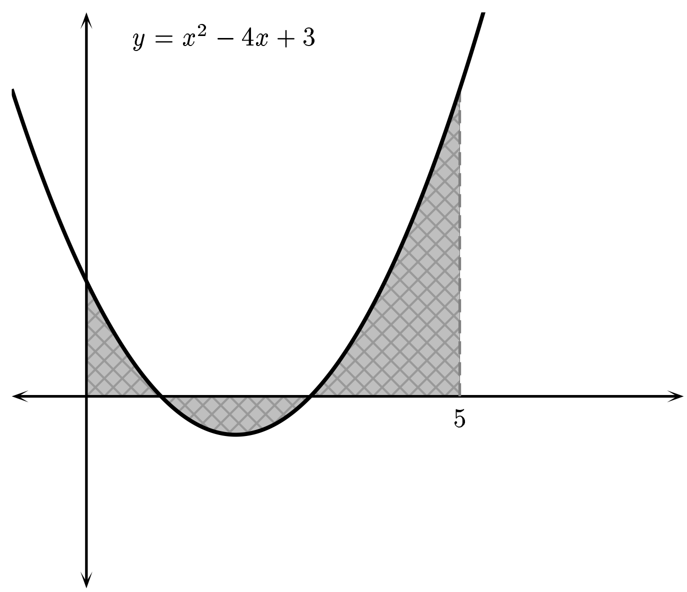
9. (8 points) Set up a Riemann sum approximation to the integral below by partitioning the interval [0, 4] into 4 subintervals of equal length and using the right end point x_k of each subinterval to calculate the height of the corresponding rectangle. *DO NOT EVALUATE THE SUM.*

$$\int_0^4 x^3\,dx$$

10. (16 points) Let $f(x) = x^2 - 4x + 3$. The graph of $y = f(x)$ is shown below.

(a) Compute $\int_0^5 f(x)\,dx$.

(b) Find the total **area** of the shaded region.



11. (18 points) Let $f(x) = \frac{x^2}{(x-3)^2}$. Answer the questions below. **Show all reasoning using the methods of calculus.**

Note: $f'(x) = \frac{-6x}{(x-3)^3}$ and $f''(x) = \frac{12x+18}{(x-3)^4}$.

(a) Find all points where f is not continuous.

(b) Find the intervals where f is increasing and the intervals where f is decreasing.

(c) Find the intervals where f is concave up and the intervals where f is concave down.

(d) Find all local extrema.

(e) Find all inflection points.

(f) Find the equation(s) of all asymptotes.

12. (16 points) Solve the initial value problem below.

$$\frac{dy}{dx} = 3\sin 2x + 6, \quad y(0) = 1$$

13. (12 points) Let $f(x) = 3x^2 + 5x - 9$.

(a) Explain why f satisfies the hypotheses of the Mean Value Theorem over the interval $[0, 3]$.

(b) Find a point $c \in (0, 3)$ such that the slope of the tangent line at $(c, f(c))$ is equal to the slope of the line containing the points $(0, -9)$ and $(3, 23)$.