# Midterm Exam 1

Math 132-06, Fall 2005

You have 50 minutes. No notes, no books, no calculators. **You must show all work to receive credit!** Good luck!

Name: Solutions

ID #: ____________________

1. ____________ (/40 points)

2. ____________ (/15 points)

3. ____________ (/25 points)

4. ____________ (/20 points)

Total ____________ (/100 points)

**1.** [40 points] Evaluate the following limits if they exist. If the limit does not exist, explain why. Justify your answers using the limit laws or facts about continuity.

**(a)** [10 points] $\displaystyle\lim_{x\to 3} \frac{x-3}{x^2-2x-3} = \lim_{x\to 3} \frac{\cancel{x-3}}{(\cancel{x-3})(x+1)}$

$$= \lim_{x\to 3} \frac{1}{x+1}$$

$$= \frac{1}{3+1}$$

$$= \boxed{\frac{1}{4}}$$

**(b)** [10 points] $\displaystyle\lim_{x\to -1} \frac{x-3}{x^2-2x-3} = \lim_{x\to -1} \frac{1}{x+1}$, as above.

This limit does not exist, because the function approaches $+\infty$ from the right and $-\infty$ from the left.

**(c)** [10 points] $\displaystyle\lim_{x\to 0^+} \frac{\sin(2\sqrt{x})}{\sqrt{x}} = \lim_{\theta\to 0^+} \frac{\sin(2\theta)}{\theta} = \lim_{\theta\to 0^+} \frac{2\sin(2\theta)}{2\theta}$

$\theta = \cancel{2}\sqrt{x}$

$$= 2\lim_{\theta\to 0^+} \frac{\sin(2\theta)}{2\theta}$$

$$= 2\cdot 1 = \boxed{2}.$$

**(d)** [10 points] $\displaystyle\lim_{x\to\infty} \frac{\sqrt{x}+5}{3\sqrt{x}-2} = \lim_{x\to\infty} \frac{\sqrt{x}/\sqrt{x} + 5/\sqrt{x}}{3\sqrt{x}/\sqrt{x} - 2/\sqrt{x}}$

$$= \lim_{x\to\infty} \frac{1+5/\sqrt{x}}{3-2/\sqrt{x}}$$

$$= \boxed{\frac{1}{3}}.$$

**2.** [15 points] Evaluate $\lim_{x \to 0^+} \sqrt{x}\cos(1/x)$, using the Sandwich Theorem.

$-1 \leq \cos(1/x) \leq 1$ for all $x$.

So $-\sqrt{x} \leq \sqrt{x}\cos(1/x) \leq \sqrt{x}$ for all $x > 0$.

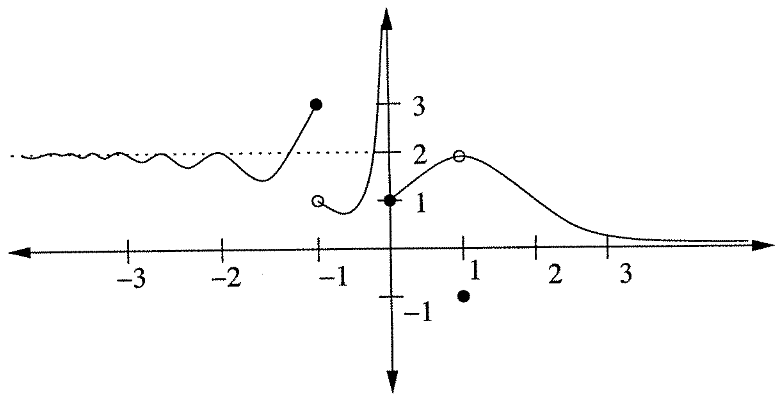
Thus, by the Sandwich Theorem,

$$\lim_{x \to 0^+} -\sqrt{x} \leq \lim_{x \to 0^+} \sqrt{x}\cos(1/x) \leq \lim_{x \to 0^+} \sqrt{x}$$

$$0 \leq \lim_{x \to 0^+} \sqrt{x}\cos(1/x) \leq 0.$$

So $\lim_{x \to 0^+} \sqrt{x}\cos(1/x) = 0$.

**3.** [25 points] This problem uses the following graph of $f(x)$.




**(a)** [9 points] Compute $\lim_{x\to 0^-} f(x)$, $\lim_{x\to -1^+} f(x)$, and $\lim_{x\to 1} f(x)$.

$\lim_{x \to 0^-} f(x) = \infty.$

$\lim_{x \to -1^+} f(x) = 1.$

$\lim_{x \to 1} f(x) = 2.$

**(b)** [6 points] At what $x$-values is $f$ discontinuous?

At $x = -1, 0,$ and $1$.

**(c)** [10 points] What are the horizontal and vertical asymptotes?

Horizontal asymptotes at $y=0, y=2$.

Vertical asymptote at $x=0$.

**4.** [20 points]

**(a)** [8 points] State the Intermediate Value Theorem.

Let $f(x)$ be a continuous function on $[a,b]$, and let $N$ be any number between $f(a)$ and $f(b)$. Then there is an x-value $c$, in the interval $[a,b]$, such that $f(c)=N$.

Decide whether each of the following statements is true or false, and explain your reasoning. **You must write out the word "True" or "False" for each one.**

**(b)** [6 points] The function $f(x) = x - \cos(x)$ has a root between $x = 0$ and $x = \frac{\pi}{2}$.

TRUE. $f(0) = 0 - \cos(0) = -1.$

$f(\pi/2) = \frac{\pi}{2} - \cos(\frac{\pi}{2}) = \frac{\pi}{2}.$

Also, $f(x)$ is continuous, because it is the difference of two continuous functions. Thus, by the Intermediate Value Theorem, $f(c)=0$ for some $c$ in $[0, \pi/2]$.

**(c)** [6 points] The function $g(x) = (x+2)\frac{|x|}{x}$ has a root between $x = -1$ and $x = 1$.

FALSE. $g(-1) = (-1+2)\cdot\frac{|-1|}{-1} = 1\cdot -1 = -1.$

$g(1) = (1+2)\cdot\frac{|1|}{1} = 3\cdot 1 = 3.$

But $g(x)$ is not continuous, so IVT doesn't apply. In fact, the only root of $g(x)$ occurs when $x+2=0$, i.e. at $x=-2$.