Midterm Exam 2

Math 132-06, Fall 2005

You have 50 minutes. No notes, no books, no calculators. **You must show all work to receive credit!** Good luck!

Name: Solutions

ID #: ____________

1. ________ (/40 points)

2. ________ (/15 points)

3. ________ (/15 points)

4. ________ (/15 points)

5. ________ (/15 points)

Total ________ (/100 points)

Test Average ________

Gateway Points ________

Course Average ________

1. [40 points] Evaluate the following derivatives. Please do not simplify your answers.

(a) [10 points] $\frac{d}{dx}\frac{2x+5}{2x-3} = \frac{(2x-3)\cdot 2 - (2x+5)\cdot 2}{(2x-3)^2}$ $\left[= \frac{-16}{(2x-3)^2} \right]$

(b) [10 points] $\frac{d}{dx}\sin\left(\sqrt{x^3+2x}\right) = \cos\left(\sqrt{x^3+2x}\right)\cdot\frac{1}{2\sqrt{x^3+2x}}\cdot(3x^2+2)$

(c) [10 points] $\frac{d}{dx}\left(3-4x^2\right)\sec(x-1) = (3-4x^2)\sec(x-1)\tan(x-1)$
$+ (-8x)\sec(x-1)$

(d) [10 points] Solve for $\frac{dy}{dx}$, where x and y are related by the equation $\tan(xy) - y\cos(x) = 1$.

$\tan(xy) - y\cos(x) = 1$

$\sec^2(xy)\left(y + x\frac{dy}{dx}\right) - \frac{dy}{dx}\cos(x) + y\sin(x) = 0$

$y\sec^2(xy) + x\frac{dy}{dx}\sec^2(xy) - \frac{dy}{dx}\cos(x) + y\sin(x) = 0$

$y\sec^2(xy) + y\sin(x) = \frac{dy}{dx}\cos(x) - \frac{dy}{dx}\cdot x\sec^2(xy)$

$\frac{dy}{dx} = \frac{y\sin(x) + y\sec^2(xy)}{\cos(x) - x\sec^2(xy)}$

2. [15 points] Let $f(x) = x^2+3x+2$. Calculate $f'(x)$ using the limit definition of the derivative. *There will be no credit given for answers obtained any other way.*

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0} \frac{(x+h)^2 + 3(x+h) + 2 - (x^2+3x+2)}{h}$$

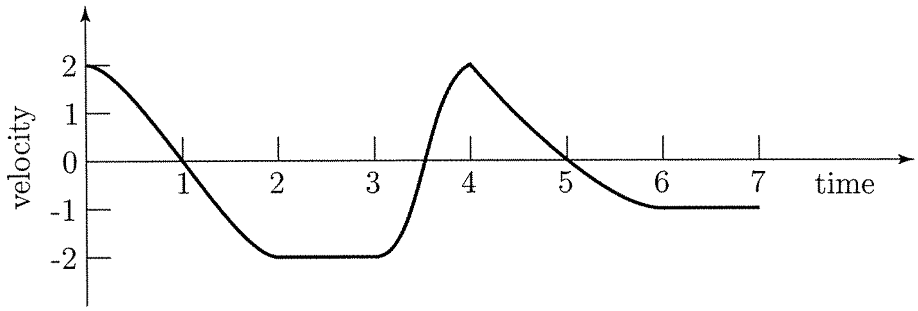
$$= \lim_{h\to 0} \frac{\cancel{x^2} + 2xh + h^2 + \cancel{3x} + 3h + \cancel{2} - \cancel{x^2} - \cancel{3x} - \cancel{2}}{h}$$

$$= \lim_{h\to 0} \frac{2xh + h^2 + 3h}{h}$$

$$= \lim_{h\to 0} 2x + h + 3$$

$$= 2x + 3.$$

3. [15 points] A particle moves along the x-axis. The **velocity** of the particle at time t is given by the graph below.



(a) [6 points] List all the time intervals when the particle is moving to the left.

(1, 3.5) and (5, 7)
(Negative velocity)

(b) [6 points] List all time intervals when the particle has positive acceleration.

(3, 4).
(Increasing velocity)

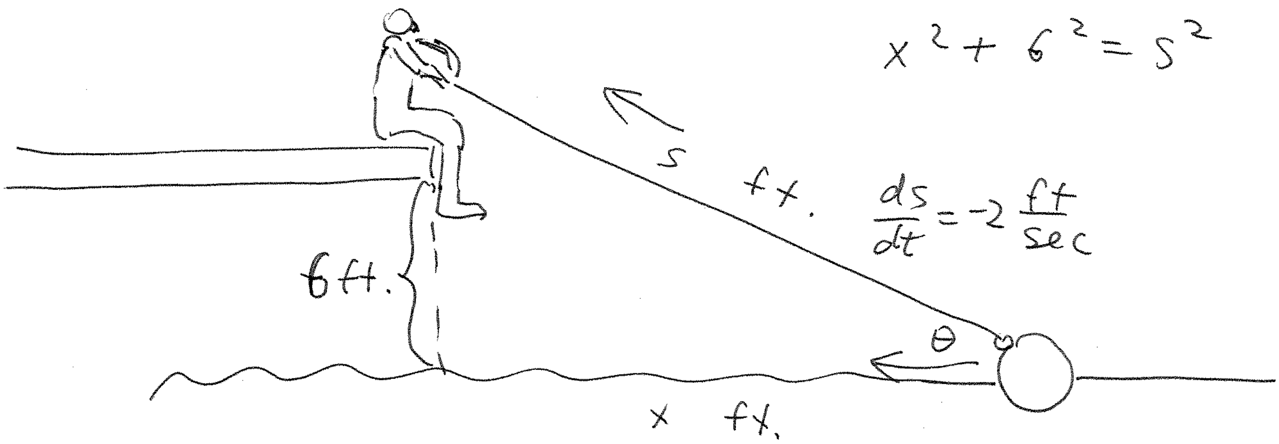
(c) [3 points] Is the particle speeding up or slowing down at $t = 4.5$ seconds?

Slowing down. (The rate of change of the speed is negative.)

4. [15 points] You are sitting on a dock, 6 feet above the water, and using a rope to pull in a floating buoy. You are pulling in the rope at a speed of 2 ft/sec.

(a) [6 points] Sketch a picture of this situation, and set up an equation that relates the distances involved.

$$x^2 + 6^2 = s^2$$



(b) [6 points] How fast is the horizontal distance between the buoy and the dock changing when the rope is 12 feet long?

$$2x \frac{dx}{dt} = 2s \frac{ds}{dt}$$

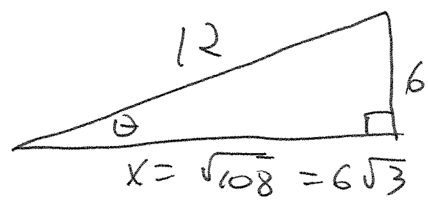
$$x \frac{dx}{dt} = s \frac{ds}{dt}$$

$$6\sqrt{3} \frac{dx}{dt} = 12(-2)$$

$$\frac{dx}{dt} = \frac{-24}{6\sqrt{3}}$$

$$\frac{dx}{dt} = \frac{-4}{\sqrt{3}} \text{ ft/sec} \left[= \frac{24}{\sqrt{108}} \frac{\text{ft}}{\text{sec}}\right].$$

lengths at this moment:



(c) [3 points] Is the buoy speeding up or slowing down at this moment? Explain.

The buoy is speeding up. There are several ways to see this.

① solve for $\frac{dx}{dt}$. $x \frac{dx}{dt} = s \frac{ds}{dt} = -2s$. $\frac{ds}{dt} = -2$, always.

So $\frac{dx}{dt} = \frac{-2s}{x}$. As the buoy gets closer, the ratio $\frac{s}{x} = \frac{1}{\cos\theta}$ gets bigger. So the speed goes up.

4 (c), continued.

(2) plug in another point!
when the rope is 10 feet long, $x=8$.

$$x \frac{dx}{dt} = -2s$$

$$8 \frac{dx}{dt} = -20$$

$$\frac{dx}{dt} = \frac{-20}{8} = -2.5 \text{ ft/sec.}$$

Because $2.5 > \frac{4}{\sqrt{3}}$, speed is going up.
(velocity gets more negative).

(3) Second derivatives.

$$\text{Acceleration} = \frac{dv}{dt} = \frac{d^2x}{dx^2}.$$

we know $x \frac{dx}{dt} = -2s$. $\Rightarrow$ $x \cdot v = -2s$

So $x \cdot \frac{dv}{dt} + \frac{dx}{dt} \cdot v = -2 \frac{ds}{dt}$

$$x \cdot a + v^2 = -2 (-2)$$

$$6\sqrt{3} \cdot a + \left(\frac{-4}{\sqrt{3}}\right)^2 = 4$$

$$6\sqrt{3} \cdot a + \frac{16}{3} = 4$$

$$6\sqrt{3} \cdot a = 4 - \frac{16}{3} < 0.$$

So $a<0$ (velocity is getting more negative).

(4) Physics. As the buoy gets closer, it takes more force to pull the rope with a constant speed. (Try it!) You are not actually pulling the buoy vertically (out of the water), so this extra force translates into horizontal acceleration.

5. [15 points] If you drop a pebble into a pond, it will make waves that become dampened with distance. At a particular moment, the height of a wave at x inches from the point of impact is

$$h(x) = \frac{\sin(x)}{x}.$$

(a) [8 points] Compute the linearization $L(x)$ of this function at $a = \frac{\pi}{2}$ inches.

$h(x) = \frac{\sin(x)}{x}$. $\quad h(a) = \frac{\sin(\pi/2)}{\pi/2} = \frac{1}{\pi/2} = \frac{2}{\pi}$.

$h'(x) = \frac{x\cos(x) - \sin(x)}{x^2}$. $\quad h'(a) = \frac{\pi/2 \cdot \cos(\pi/2) - \sin(\pi/2)}{(\pi/2)^2}$

$= \frac{-1}{(\pi/2)^2} = \frac{-4}{\pi^2}$.

$L(x) = h(a) + h'(a)(x-a)$

$= \frac{2}{\pi} + \frac{-4}{\pi^2}\left(x - \frac{\pi}{2}\right)$.

(b) [4 points] Use $L(x)$ to estimate the height of the wave at π inches from the point of impact.

$L(\pi) = \frac{2}{\pi} - \frac{4}{\pi^2}\left(\pi - \frac{\pi}{2}\right)$

$= \frac{2}{\pi} - \frac{4}{\pi^2}\left(\frac{\pi}{2}\right)$

$= \frac{2}{\pi} - \frac{2}{\pi}$

$= 0$.

(c) [3 points] Compare the estimate from part (b) to the exact value $h(\pi)$. What can you say about this approximation?

$h(\pi) = \frac{\sin(\pi)}{\pi} = \frac{0}{\pi} = 0$.

So the approximation gives the exact value.