Midterm Exam 4

Math 132-06, Fall 2005

You have 50 minutes. No notes, no books, no calculators. **You must show all work to receive credit!** Good luck!

Name: Solutions

ID #: ____________________

1. ________ (/40 points)

2. ________ (/15 points)

3. ________ (/15 points)

4. ________ (/15 points)

5. ________ (/15 points)

Total ________ (/100 points)

Test Average ________

Gateway Points ________

Course Average ________

1. [40 points] Compute the following definite and indefinite integrals.

(a) [10 points] $\int \frac{x^2-2}{\sqrt{x}}\,dx$

$$= \int \frac{x^2}{\sqrt{x}} - \frac{2}{\sqrt{x}}\,dx$$

$$= \int x^{3/2} - 2x^{-1/2}\,dx$$

$$= \frac{2}{5}x^{5/2} - 4x^{1/2} + C$$

(b) [10 points] $\int \frac{x^2}{(16-x^3)^2}\,dx$

$$= \int \frac{1}{-3u^2}\,du$$

$u = 16 - x^3$

$du = -3x^2\,dx$

$\frac{du}{-3} = x^2\,dx$

$$= \frac{1}{-3} \cdot -u^{-1} + C$$

$$= \frac{1}{3(16-x^3)} + C$$

(c) [10 points] $\int_0^{\frac{\pi}{4}} \sqrt{\tan x}\,\sec^2 x\,dx$

$$= \int_0^1 \sqrt{u}\,du$$

$u = \tan x$

$du = \sec^2 x$

$x = 0 \to u = 0$

$x = \frac{\pi}{4} \to u = 1$

$$= \frac{2}{3}u^{3/2}\Big]_0^1$$

$$= \frac{2}{3}(1) - \frac{2}{3}(0)$$

$$= \frac{2}{3}.$$

(d) [10 points] $\int_{-1}^{1} |x| + x^2\,dx$

$$= \int_{-1}^{0} -x + x^2\,dx + \int_0^1 x + x^2\,dx$$

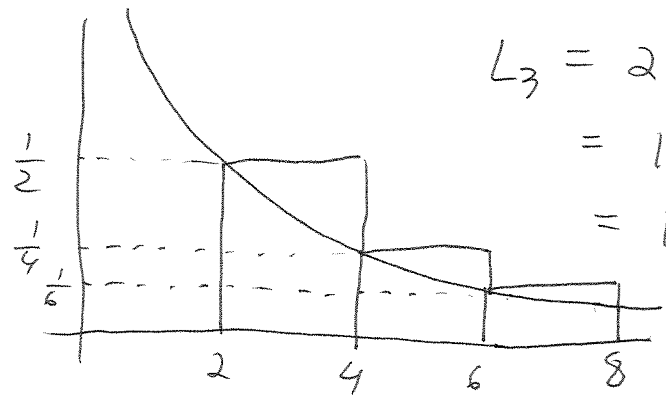
$$= -\frac{x^2}{2} + \frac{x^3}{3}\Big]_{-1}^{0} + \left[\frac{x^2}{2} + \frac{x^3}{3}\right]_0^1$$

$$= \left(\frac{0}{2} + \frac{0}{3}\right) - \left(\frac{-1}{2} + \frac{-1}{3}\right) + \left(\frac{1}{2} + \frac{1}{3}\right) - (0+0)$$

$$= \frac{1}{2} + \frac{1}{3} + \frac{1}{2} + \frac{1}{3} = 1\frac{2}{3}.$$

2. [12 points] Consider the function $f(x) = \dfrac{1}{x}$ on the interval $[2, 8]$.

(b) [9 points] Compute L_3, the left-hand estimate for $\displaystyle\int_2^8 \frac{1}{x}\,dx$ using 3 rectangles.

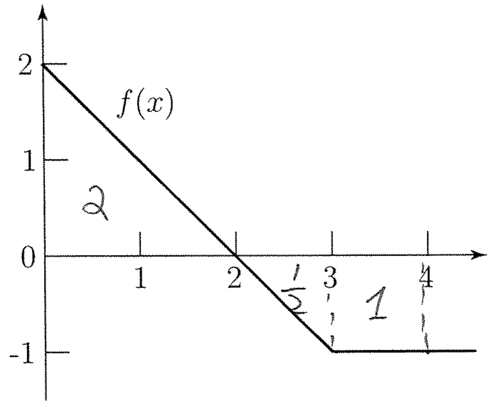


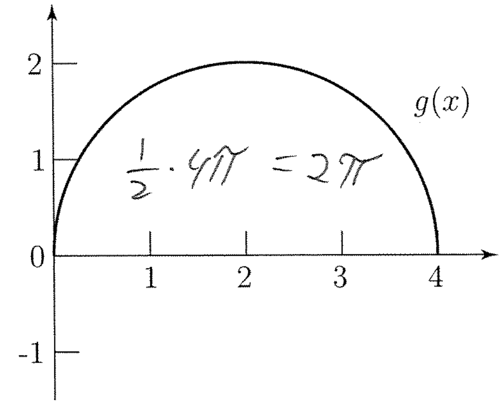
$$\begin{aligned} L_3 &= 2 \cdot \frac{1}{2} + 2 \cdot \frac{1}{4} + 2 \cdot \frac{1}{6} \\ &= 1 + \frac{1}{2} + \frac{1}{3} \\ &= 1\frac{5}{6}. \end{aligned}$$

(b) [6 points] Is this an over- or under-estimate? Explain.

This is an over-estimate. The function $f(x) = \frac{1}{x}$ is decreasing, so the rectangles from the left end point approximation are taller than the actual function.

3. [15 points] Below are the graphs of two functions, f and g.





Compute the following definite integrals.

(a) [5 points] $\int_0^4 f(x) + 2g(x)\,dx$ $= \int_0^4 f(x)\,dx + 2\int_0^4 g(x)\,dx$

$= 2 - \frac{1}{2} - 1 + 2 \cdot 2\pi$

$= \frac{1}{2} + 4\pi.$

(b) [5 points] $\int_2^1 f(x)\,dx$ $= -\int_1^2 f(x)\,dx$

$= -\frac{1}{2}.$

(c) [5 points] $\int_1^1 f(x)\,g(x)\,dx$ $= 0.$

$\int_a^a (\text{any function})\,dx = 0.$

4. [15 points] Compute the derivatives of the following functions. You don't have to simplify.

(a) [8 points] $f(x) = \int_1^x (t^2 - 3t)^5 \, dt$

$$f'(x) = (x^2 - 3x)^5$$

u

(b) [7 points] $g(x) = \int_0^{\sin x} t \cos(t) \, dt$

$$\frac{dg}{du} = u \cdot \cos(u). \qquad \frac{du}{dx} = \cos(x)$$

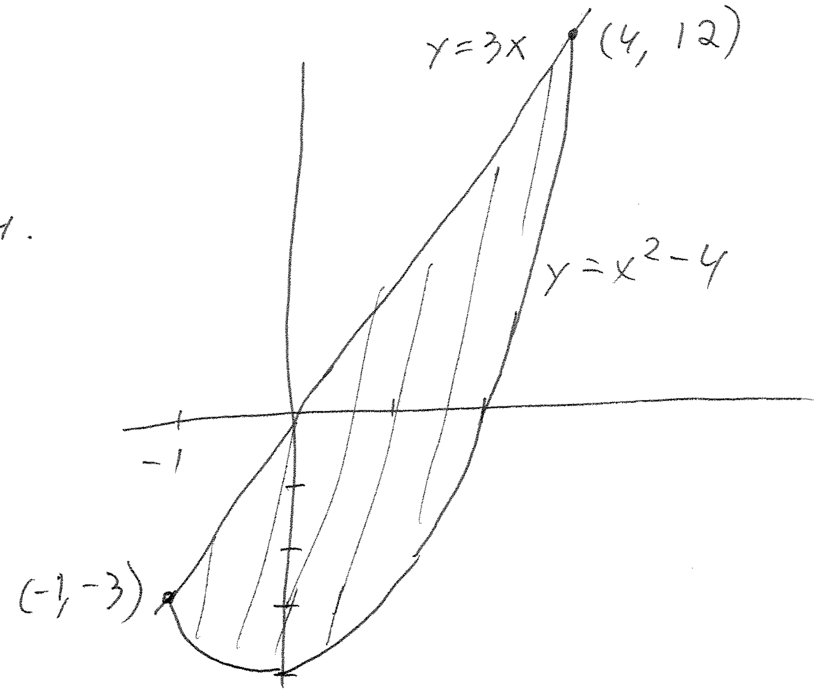
$$\frac{dg}{dx} = \frac{dg}{du} \cdot \frac{du}{dx} = u \cos(u) \cdot \cos(x)$$

$$= \sin(x) \cdot \cos(\sin x) \cdot \cos(x).$$

5. [15 points] Let R be the region enclosed between the graphs of $y = 3x$ and $y = x^2 - 4$.

(a) [7 points] Sketch a graph of R and label the points of intersection of the two curves.

$$3x = x^2 - 4$$
$$x^2 - 3x - 4 = 0$$
$$(x+1)(x-4) = 0$$
$$x = -1 \quad \text{or} \quad x = 4.$$



(b) [8 points] What is the area of R?

$$A = \int_{-1}^{4} 3x - (x^2 - 4)\,dx$$
$$= \int_{-1}^{4} 3x - x^2 + 4\,dx$$
$$= \left[\frac{3}{2}x^2 - \frac{1}{3}x^3 + 4x\right]_{-1}^{4}$$
$$= \left[\frac{3}{2}(16) - \frac{1}{3}(64) + 16\right] - \left[\frac{3}{2}(+1) - \frac{1}{3}(-1) + 4(-1)\right]$$
$$= 24 - \frac{64}{3} + 16 - \frac{3}{2} - \frac{1}{3} + 4$$
$$= 44 - \frac{65}{3} - \frac{3}{2}$$
$$= 20\frac{5}{6} \quad \left(= \frac{125}{6}\right)$$