

MATH 310: EXTRA CREDIT PROBLEMS

(1) (10 points, due 8/12) For which primes p is the polynomial $x^2 + 1$ irreducible in $\mathbb{Z}_p[x]$? Equivalently, when does the equation $x^2 = -1$ have no solutions mod p ? You do not have to prove anything. Just do some examples and write down what you think the pattern is.

(2) (10 points, due 8/19) There are 14 ways to multiply 5 given elements of a group. For example: $(ab)((cd)e)$. The associativity law can be applied in three ways to each such arrangement of parentheses.

Draw (or build) the following 3D polyhedron: the vertices correspond to the 14 ways of combining 5 elements and the edges correspond to the ways the associativity law can be applied to transform one arrangement of parentheses into another. You should end up with 21 edges.

(3) (10 points, due 8/19) There are 24 ways to permute four letters a, b, c, d . For each permutation, we can produce three new permutations by swapping the first two letters, the middle two letters, or the last two letters.

Draw (or build) the following 3D polyhedron: the vertices correspond to the 24 permutations of 4 letters and the edges correspond to the ways a permutation can be transformed into another using the three swaps described above. You should end up with 36 edges.