

## MATH 310: HOMEWORK 10

(1) Let  $R$  be an integral domain which is not a field. Use the first isomorphism theorem to prove that  $(x)$  is a non-zero prime ideal in  $R[x]$ , but not a maximal ideal.

(2) Find all the generators of  $\mathbb{Z}_{23}^\times$ .

Hint: After you find one generator, the rest are easy to find.

(3) Find all the generators of  $(\mathbb{Z}_5[x]/(x^2 - 2))^\times$ .

Hint: First find all the powers of  $x$ .

(4) Let  $\mathbb{R}$  denote the group of real numbers (under addition) and let  $\mathbb{R}_{>0}$  denote the group of positive real numbers (under multiplication). Prove that  $\mathbb{R}$  and  $\mathbb{R}_{>0}$  are isomorphic groups.

Hint: Find a one-to-one function  $f$  whose domain is  $\mathbb{R}$ , whose range is  $\mathbb{R}_{>0}$ , and  $f(a + b) = f(a)f(b)$  for any  $a, b \in \mathbb{R}$ .

(5) Let  $A$  be a commutative ring with identity. Consider the determinant function:

$$\det : GL(2, A) \rightarrow A^\times$$
$$\det \begin{pmatrix} a & b \\ c & d \end{pmatrix} = ad - bc$$

where  $a, b, c, d \in A$ . Prove that  $\det$  is a group homomorphism. What is the kernel of  $\det$ ?