MATH 481
REVIEW PROBLEMS

1. How many possible outcomes are there if we roll a die fifty times? How about if we roll fifty dice at the same time?

2. Count the number of possibilities for each of the following situations:

   (a) The number of passwords of length 6-8, using an alphabet of 26 letters.

   (b) The number of passwords of length 6-8, using an alphabet of 26 letters, with no repeated characters.

   (c) The number of ways to rearrange the letters of the word *rearrange*.

   (d) The number of ways to order five scoops of ice cream, choosing from three given flavors.

   (e) The number of ways to flip a coin ten times and get at least six tails.

   (f) The number of 3-cycles in K_n.

3. Determine whether each of the following statements is true or false. Justify your answer with a short explanation.

   (a) Any subgraph of $K_{3,3}$ or K_5 is non-planar.

   (b) A graph of order n and size n must be connected.

   (c) Let G be a graph. If $\chi(G) \leq 4$, then G is planar.

   (d) Every tree is bipartite.

   (e) There is a polyhedron with 28 triangular faces and 17 pentagonal faces.

   (f) The Petersen graph does not have an Eulerian circuit.

4. Prove that

$$\sum_{k=0}^{n} \binom{n}{k}^2 = \binom{2n}{n}$$

   Hint: $(1+x)^{2n} = (1+x)^n(1+x)^n$.

5. How many eight-slice pizzas are there if each slice is allowed any to have any combination of two given toppings?

6. Consider the following recurrence: $a_0 = a_1 = 0, a_2 = 1$, and $a_k = a_{k-1} + a_{k-2} + a_{k-3}$. Find a closed form for the generating function.

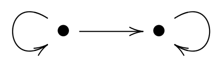
7. Find the coefficient of x^{10k} in

$$A(x) = \frac{1}{(1-x)(1-x^5)(1-x^{10})}$$

Hint: the answer is $(k+1)^2$.

8. Prove that a connected graph of order n and size n contains exactly one cycle.

9. Consider the following digraph:



Use the adjacency matrix to find the number of walks of length n from one vertex to the other.

10. Draw all the unlabeled trees of order 7, and find the number of symmetries of each one.

Hint: you can check your work by using this formula:

$$n^{n-2} = n! \sum_T \frac{1}{s(T)}$$

The sum runs over unlabeled trees of order n, and $s(T)$ is the number of symmetries of a given tree T.