MATH 481: EXTRA CREDIT PROBLEMS

These extra credit problems are due on the last day of class (12/11).

(1) Give a combinatorial proof of:

$$\sum_{k \text{ odd}}^{n} \binom{n}{k} = \sum_{k \text{ even}}^{n} \binom{n}{k}$$

(2) Draw the following polyhedron (as a connected planar graph). The vertices are permutations of the four letters a, b, c, d. Two permutations are adjacent if we can get from one to other by swapping the first two entries, or the middle two entries, or the last two entries.

(3) Draw the following polyhedron (as a connected planar graph). The vertices correspond to the triangulations of a regular hexagon. Two triangulations are adjacent if they are related by swapping the diagonal of one of the three quadrilaterals in the triangulation.

The next two problems are quite difficult. You should try them if you enjoy a challenge.

(4) Let A_n be the number of possible outcomes in a bracket for a single-elimination tournament with $n+1$ rounds. Find a recursive formula for A_n. Hint: A_n is equal to the number of ways to assign the numbers 0 and 1 to the vertices of a complete binary tree of height n so that a parent assigned 1 cannot have both children assigned 0.

(5) Give a combinatorial proof of:

$$6\sum_{k=1}^{n} k^2 = n(n+1)(2n+1)$$

Hint: First try to prove:

$$6\binom{n+2}{3} = n(n+1)(n+2)$$