

## MATH 481: HOMEWORK 10

- (1) Determine whether each of the following statements is true or false. Justify your answer.
  - (a) Every bipartite graph is planar.
  - (b) Every planar graph is bipartite.
  - (c) A tree whose size is even must have a vertex of even degree.
  - (d)  $L(\overline{C_6})$  is a 3-colorable polyhedron.
- (2) Calculate each of the following:
  - (a) The number of 4-cycles in  $K_{n,m}$ .
  - (b) The number of 5-cycles in  $K_{n,m}$ .
  - (c) The order and size of a polyhedron with  $r$  triangular faces.
  - (d) The chromatic polynomial of  $K_n - e$  (the graph of order  $n$  and size  $\binom{n}{2} - 1$ ).
- (3) Draw all the unlabeled graphs of order 5 and size 4. Find the number of symmetries and labelings of each graph, and check that all the labelings add up to:

$$\binom{5}{4}$$

- (4) Draw all the unlabeled trees of order 6. Find the number of symmetries and labelings of each tree, and check that all the labelings add up to  $6^{6-2}$ .
- (5) Let  $g(n, k)$  denote the number of connected labeled graphs of order  $n$  and size  $k$ . Prove that:

$$\sum_{k=n-1}^{\binom{n}{2}} (-1)^k g(n, k) = (-1)^{n-1} (n-1)!$$