

**MATH 481: HOMEWORK 9**

- (1) (a) Find a disconnected graph whose order and size are equal.  
 (b) Find a connected graph that is not complete and not bipartite.
- (2) Prove that the number of ways to choose  $k$  edges from a tree of order  $n$  is equal to the number of ways to put  $k$  identical objects into  $n - k$  bins.
- (3) Give an algebraic proof of the following:

$$\binom{2n}{3} = 2\binom{n}{3} + 2n\binom{n}{2}$$

Now give a combinatorial proof using 3-cycles in  $\overline{K_{n,n}}$ .

- (4) Find the number of 4-cycles in  $K_{n,m}$ .
- (5) A polyhedron with  $n$  vertices has one pentagonal face and the rest of its faces are triangles. How many edges and faces does it have? State your answer in terms of  $n$ .
- (6) Find  $\chi(L(K_5))$ .
- (7) Find the chromatic polynomial of  $\overline{C_6}$ .
- (8) Let  $T$  be a labeled tree of order  $n$ . Suppose  $T$  has a vertex of degree  $m$  and another vertex of degree  $n - m$ . Prove that the number of such trees is:

$$\binom{n}{2} \binom{n-2}{m-1}$$

- (9) Let  $a_0 = 0, a_1 = 3$  and  $a_k = 2a_{k-1} - a_{k-2}$  for  $k \geq 2$ . Write the generating function as a quotient of two polynomials and find an explicit formula for  $a_k$ .
- (10) Think of  $K_{1,6}$  as the rigid planar figure given below. Find the number of proper 3-colorings of this graph, taking the six rotations into account.

