

### MATH 481: REVIEW 3

- (1) A graph is called *k-regular* if  $\deg(v) = k$  for any vertex  $v$ . What is the size of a  $k$ -regular graph of order  $n$ ?
- (2) Let  $G$  be a graph. The *complement*  $\overline{G}$  is a graph with the same set of vertices as  $G$ , but two vertices are adjacent in  $\overline{G}$  if and only if they are *not* adjacent in  $G$ . Prove that:

$$\text{The size of } \overline{K_{m,n}} \text{ is } \binom{n}{2} + \binom{m}{2}$$

- (3) Let  $T$  be a tree with an even number of edges. Prove that  $T$  must have a vertex of even degree.
- (4) Let  $T$  be a labeled tree without any repeated entries in its Prüfer code. Prove that  $T$  is a path.
- (5) Let  $G$  be a planar graph with  $k$  connected components. Prove that

$$n - q + r = 1 + k$$

- (6) Suppose a polyhedron has  $r$  faces, all of them triangles. How many edges and vertices does it have?
- (7) Let  $G$  be a graph of order  $n$  and size  $q$ . The *cone*  $C(G)$  is a graph with all the edges and vertices of  $G$ , but with one more vertex that is adjacent to all the vertices of  $G$ . Prove that  $C(G)$  has order  $n + 1$  and size  $q + n$ . Also, prove that:

$$\chi(C(G)) = \chi(G) + 1$$