MATH 482: HOMEWORK 1

(1) Write down the addition and multiplication tables for $\mathbb{Z}_8$. Are they Latin squares? If not, find the Latin square obtained by removing all the rows and columns with repeated entries.

(2) For each element $\overline{a} \in \mathbb{Z}_{12}$, find the smallest positive integer k such that $\overline{ka} = \overline{0}$. Make a table of these values. How would you describe the pattern?

(3) Find $(1! + 2! + 3! + 4! + 5! + \cdots + 100!) \bmod 100$.

(4) Determine whether $\sqrt{-1}$ exists in $\mathbb{Z}_p$ for $p = 3$, 5, 7, 11, 13, and 17. Can you find a simple rule for determining whether $\sqrt{-1}$ exists in $\mathbb{Z}_p$, where p is an odd prime number?

Hint: For the first part, notice that you only have to check those $\overline{k}$ satisfying $k^2 \geq p - 1$ and $k < p/2$, because of symmetry. For the second part, your answer should involve $p - 1$. You do not have to prove anything, but you should do enough examples to notice the pattern.

(5) Determine whether $\sqrt{2}$ exists in $\mathbb{Z}_p$ for $p = 3$, 5, 7, 11, 13, and 17. Can you find a simple rule for determining whether $\sqrt{2}$ exists in $\mathbb{Z}_p$, where p is an odd prime number?

Hint: The pattern is similar to the one from number (4), but it involves $p - 1$ and $p + 1$.