MATH 482: HOMEWORK 10

(1) Let n be a positive integer. Let $S(n)$ be the number of self-conjugate partitions of n. Let $O(n)$ be the number of partitions of n which consist of distinct odd numbers. Prove that $S(n) = O(n)$.

Hint: Give a combinatorial proof via Young diagrams.

(2) Let $Q(n,k)$ be the number of *ordered* partitions of n into k parts, i.e. the number of sequences of positive integers $(a_1, \dots, a_k)$ so that $a_1 + \cdots + a_k = n$. Let $Q(n)$ be the total number of ordered partitions of n. Prove that:

$$Q(n,k) = \binom{n-1}{k-1} \qquad \text{and} \qquad Q(n) = 2^{n-1}$$

(3) Define the following set of n-tuples of non-negative integers:

$$S = \{(s_1, \dots, s_n) \mid \sum_{k=1}^{n} k s_k = n\}$$

Prove the following formula:

$$\sum_{(s_1,\dots,s_n)\in S} \frac{(s_1 + s_2 + \cdots + s_n)!}{s_1! s_2! \dots s_n!} = 2^{n-1}$$

(4) Prove the following identity:

$$\binom{n}{k}_q = q^k \binom{n-1}{k}_q + \binom{n-1}{k-1}_q$$

(5) For $0 \le r \le n$, find an explicit formula for the coefficient of q^r in:

$$\binom{n+2}{2}_q$$