

1 Applications of Functions to Economics

* The Cost Function

The **cost function**, $C(q)$, gives the total cost of producing a **quantity** q of some good.

$$\text{The total costs} = \text{Fixed Costs} + \text{Variable Costs},$$

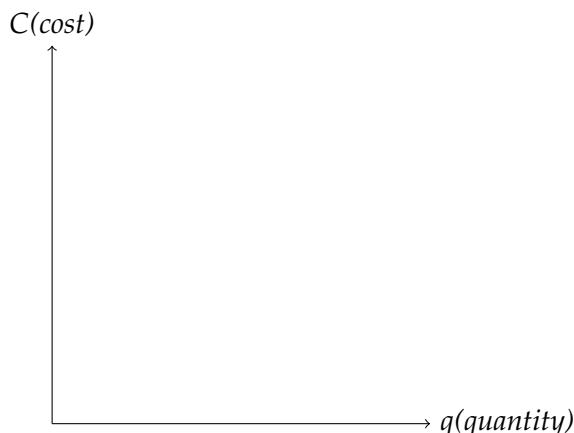
where **Fixed Costs** are incurred even if nothing is produced and **Variable Costs** depend on how many units are produced.

If $C(q)$ is a linear cost function,

Fixed costs are represented by the vertical intercept.

Marginal cost is represented by the slope.

Example 1 A company produces and sells shirts. The fixed costs are \$7000 and the variable costs are \$5 per shirt. Find a formula for the cost function $C(q)$ as a function of the quantity of shirts, q . And graph the cost function.



* The Revenue Function

The **revenue function**, $R(q)$, gives the total revenue received from a firm from selling a **quantity**, q , of some good. If the good sells for a price of p per unit, then

$$\text{Revenue} = \text{Price} * \text{Quantity},$$

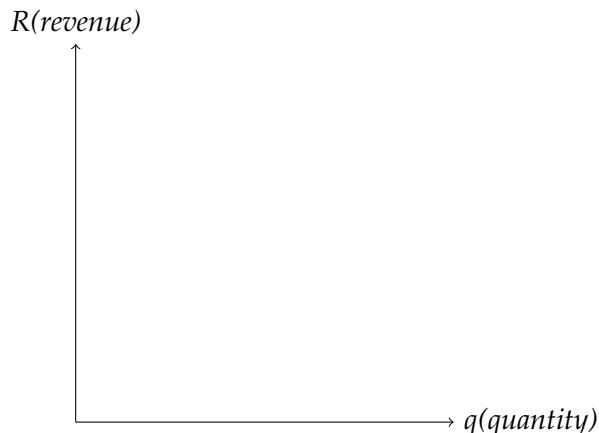
which is exactly the same as

$$R = pq.$$

If the price does not depend on the quantity sold, so p is a constant, the graph of revenue as a function of q is a line through the origin, with slope equal to the price p .

The **marginal revenue** is also represented by the slope.

Example 2 In Example 1, if the shirts are sold for \$12 each, find a formula for the revenue function $R(q)$ as a function of the quantity of shirts, q . And graph the revenue function.



* The Profit Function

$$\text{Profit} = \text{Revenue} - \text{Cost}.$$

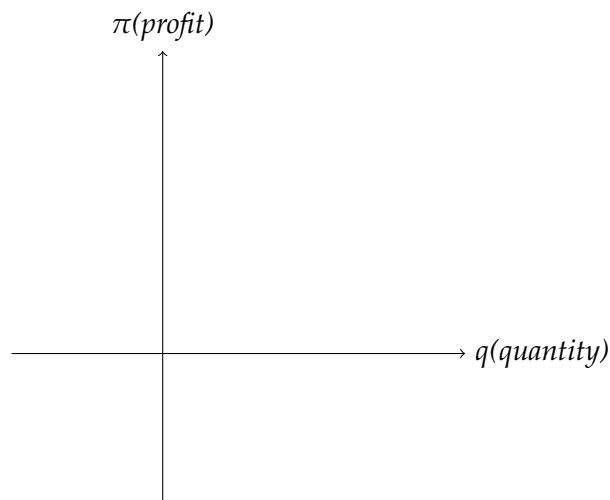
Let π denote the profit, then

$$\pi = R - C.$$

The **break-even** point is the point where the profit is zero, or equivalently, revenue equals cost.

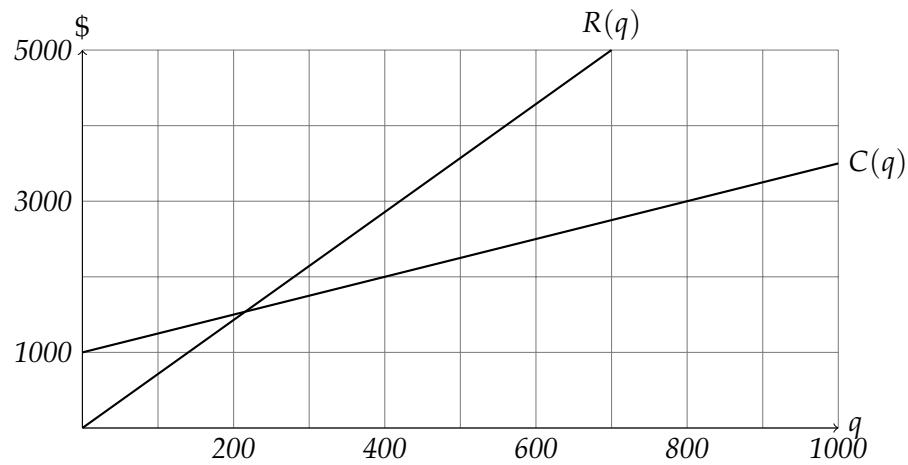
If the profit function is a linear function, then the **marginal profit** is represented by the slope.

Example 3 In Example 1 and Example 2, find a formula for the profit function $\pi(q)$ as a function of the quantity of shirts, q . Graph it and mark the break-even point.



Example 4 The following figure shows cost and revenue for a company.

- (a) Find the fixed costs and the marginal cost for the cost function $C(q)$.
- (b) Find a formula for $C(q)$.
- (c) What is the marginal revenue?
- (d) Find a formula for $R(q)$.
- (e) Approximately what quantity does this company have to produce to make a profit?
- (f) Estimate the profit generated by 500 units.



Example 5 The following table shows a company's estimates of cost and revenue for a product.

q	0	10	20	30	40
$C(q)$	500	600	700	800	900
$R(q)$	0	250	500	750	1000

- (a) What are the fixed costs and the marginal cost?
- (b) What price does the company charge for its products?
- (c) Find a formula for $C(q)$ and $R(q)$.
- (d) Find the break-even quantity.

Example 6 A company that makes Adirondack chairs has fixed costs of \$5000 and variable costs of \$30 per chair. The company sells the chairs for \$50 each.

- (a) Find a formula for the cost function.
- (b) Find a formula for the revenue function.
- (c) Find the break-even point.