

Section 5.3

- 5.39** Prove the following statement using more than one method of proof.
For every three integers a, b and c , exactly two of the integers ab, ac and bc cannot be odd.

Section 5.4

- 5.40** Show that there exist a rational number a and irrational number b such that a^b is rational.
- 5.47** Let S be a set of three integers. For a nonempty subset A of S , let σ_A be the sum of the elements in A . Prove that there exist two distinct nonempty subsets B and C of S such that $\sigma_B \equiv \sigma_C \pmod{6}$.

Section 5.5

- 5.50** Disprove the statement: There is a real number x such that $x^6 + x^4 + 1 = 2x^2$.
- 5.52** The integers 1, 2, 3 have the property that each divides the sum of the other two. Indeed, for each positive integer a , the integers $a, 2a, 3a$ have the property that each divides the sum of the other two. Show that the following statement is false.
There exists an example of three distinct positive integers different from $a, 2a, 3a$ for some $a \in \mathbb{N}$ having the property that each divides the sum of the other two.

Section 5.6

- 5.57** The king's daughter had three suitors and couldn't decide which one to marry. So, the king said, "I have three gold crowns and two silver ones. I will put either a gold or silver crown on each of your heads. The suitor who can tell me which crown he has will marry my daughter." The first suitor looked around and said that he could not tell. The second one did the same. The third suitor said: "I have a gold crown." He is correct, but the daughter was puzzled: The suitor was blind. How did he know?
- 5.62** Let a_1, a_2, \dots, a_r be odd integers where $a_i > 1$ for $i = 1, 2, \dots, r$. Prove that if $n = a_1 a_2 \dots a_r + 2$, then $a_i \nmid n$ for each integer i ($1 \leq i \leq r$).
- 2.65** Prove that there exist four distinct real numbers a, b, c, d such that exactly four of the numbers ab, ac, ad, bc, bd, cd are irrational.

Additional Exercises

- Prove or disprove: For every $x \in \mathbb{R}^+$, $x \leq x^2$.
- Prove that for every $n \in \mathbb{N}$, the equation $x^{2n+1} + x = 0$ has exactly one real solution.