
Section 5.1

5.3 Disprove the statement: If $n \in [1, 2, 3, 4, 5]$, then $3|(2n^2 + 1)$.

5.6 Let $a, b \in \mathbb{Z}$. Disprove the statement: For every two positive integers a and b, $(a + b)^3 = a^3 + 2a^2b + 2ab + 2ab^2 + b^3$.

5.8 For positive real numbers a and b, it can be shown that $(a + b)(1/a + 1/b) \geq 4$. Does it therefore follow that $(c^2 + d^2)(1/c^2 + 1/d^2) \geq 4$ for every two positive numbers c and d?

Section 5.2

5.11 Prove that there is no smallest positive irrational number.

5.19 Prove that $\sqrt{3}$ is irrational. [Hint: First prove for an integer a that $3|a^2$ if and only if $3|a$. Recall that every integer can be written as $3q, 3q + 1, or 3q + 2$ for some integer q.

5.31 Use a proof by contradiction to prove the following. Let $m \in \mathbb{Z}$. If $3 \nmid (m^2 - 1)$, then $3|m$.