
(11.4) Prove that $3|(n^3 - n)$ for every integer n.

(11.6) Find all primes that are 1 less than a perfect cube.

(11.10) (a) Prove that $6|(5n^3 + 7n)$ for every positive integer n.

(b) Observe that $5 + 7 = 12$ is a multiple of 6. State and prove a generalization of the problem in (a).

(11.14) For an integer $n \geq 2$, let m be the largest positive integer less than n such that $m|n$. Then $n = mk$ for some positive integer k. Prove that k is a prime.

(11.18) Show that, except for 2 and 5, every prime can be expressed as $10k + 1$, $10k + 3$, $10k + 7$, or $10k + 9$, where $k \in \mathbb{Z}$.

(11.22) (a) Prove that the square of every integer that is not a multiple of 3 is of the form $3k + 1$, where $k \in \mathbb{Z}$.

(b) Prove that the square of no integer is of the form $3m - 1$, where $m \in \mathbb{Z}$.

(11.28) Prove for every positive integer n that $n^2 + 1$ is not a multiple of 6.

(11.32) Give an example of a set S of four (distinct) positive integers such that the greatest common divisor of all six pairs of elements of S is 6.

(11.34) Prove for $a \in \mathbb{Z}$ and $n \in \mathbb{N}$ that $\gcd(a, a + n)|n$.

(11.36) For positive integers a, b and c, the **greatest common divisor**, $\gcd(a, b, c)$, of a, b and c is the largest positive integer that divides all of a, b and c. Let $d = \gcd(a, b, c)$, $e = \gcd(a, b)$ and $f = \gcd(e, c)$. Prove that $d = f$.

(11.37) Use the Euclidean Algorithm to find the greatest common divisor for each of the following pairs of integers:

(a) 51 and 288

(b) 357 and 629

(c) 180 and 252

(11.38) Determine integers x and y such that (see the previous exercise):

(a) $\gcd(51,\ 288) = 51x + 288y$

(b) $\gcd(357,\ 629) = 357x + 629\ y$

(c) $\gcd(180,\ 252) = 180x + 252\ y$.

(11.42) Let $a, b \in \mathbb{Z}$, where one of a, b is nonzero. Prove that if $d = \gcd(a, b)$, $a = a_1 d$ and $b = b_1 d$, then $\gcd(a_1, b_1) = 1$.