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1. (4+8+8=20pts)

(I)

+2 +2 +2 +2

x	1	3	5	7	9
f(x)	9.5	15	20.5	26	31.5

+5.5 +5.5 +5.5 +5.5

(II)

+3 +3 +3 +3

x	0	3	6	9	12
g(x)	5	10	20	40	80

·2 ·2 ·2 ·2

(III)

+1 +1 +1 +1

x	0	1	2	3	4
h(x)	5	10	15	30	40

+5 +10 +15 +20 not linear

or: ·2 ·1.5 ·2 $\cdot\frac{4}{3}$ not exponential

(a) Determine whether each of the following tables could correspond to a linear function, an exponential function, or neither.

(I): linear $m = \frac{5.5}{2} = 2.75$

(II): exponential $a^3 = 2$

(III): neither

(b) Find an explicit formula for the linear function.

$$f(x) = b + mx = b + 2.75x = 6.75 + 2.75x$$

$f(1) = 9.5 = b + 2.75 \cdot 1$

$6.75 = b$

verify: $f(6) = 6.75 + 2.75 \cdot 6 = 26$ ✓

(c) Find an explicit formula for the exponential function.

$P_0 = 5$ (vertical intercept)

$g(x) = P_0 \cdot a^x$

$a^3 = 2$

$a = \sqrt[3]{2} = 2\,\hat{}\,(1/3) = 1.2599$

$$g(x) = 5 \cdot (1.2599)^x$$

2. (5+5+5=15pts) A piece of ice that has initial mass 10 grams is taken out of the freezer and is melting at a continuous rate of 1.5% per minute.

a) Write down a formula for that describes the mass of the ice as a function of time.

$$P(t) = 10 \cdot e^{-.015t}$$

b) What is the half-life of the ice?

solve $P(t) = \frac{1}{2} \cdot P_0 = 5$

$$10e^{-.015t} = 5$$

$$e^{-.015t} = \frac{5}{10} = .5$$

$$\boxed{\ln(A^p) = p\ln(A)}$$

$$\ln(e^{-.015t}) = \ln(.5)$$

$$-.015t \ln(e) = \ln(.5)$$

$$t = \frac{\ln(.5)}{-.015} = 46.21 \text{ minutes}$$

c) When will only 1 gram of the ice remain?

solve $P(t) = 1$

$$10e^{-.015t} = 1$$

natural logarithm of both sides

$$e^{-.015t} = \frac{1}{10} = .1$$

$$-.015t = \ln(.1)$$

$$t = \frac{\ln(.1)}{-.015} = 153.51 \text{ mins}$$

3. (6+6=12pts) You deposit \$10,000 into an interest bearing account at a rate r = 5.24%.
a) Write the account balance, as a function of t years, if the interest is compounded annually with rate r.

$$P(t) = 10000\,(1 + .0524)^t$$
$$= 10000\,(1.0524)^t$$

b) Now write the account balance, as a function of t years, if the interest is compounded continuously with rate r.

$$P(t) = 10000\, e^{.0524t}$$

4. (4+4+4+4=16pts)Use the following table.

x	0	1	2	3	4
y(x)	16	15	12	7	0

a) Is the function increasing or decreasing?

b) What is the change in the value of the function between x=1 and x=3?

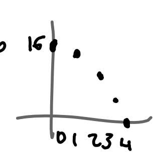
$$\Delta y = y(3) - y(1) = 7 - 16 = -8$$

c) What is the average rate of change of the function between x=1 and x=3?

$$\frac{\Delta y}{\Delta x} = \frac{7-16}{3-1} = \frac{-8}{2} = -4$$

d) Is the function concave up or down?

because ave. r.o.c. is getting more and more negative.



5. (4+4+4+4+4=20 pts) The fixed cost and marginal cost for a company which makes soccer balls are \$6000 and \$10/ball respectively. The company sells each ball at a price \$25.

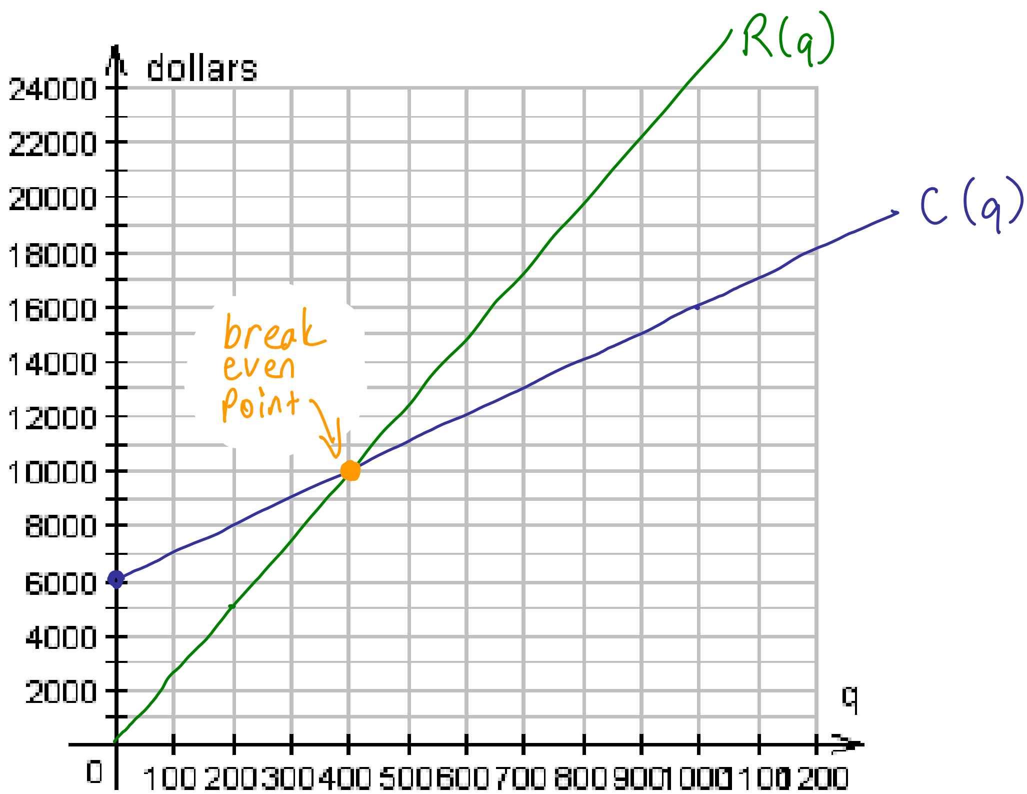
a) Write down the formula for the cost function C(q).

$$C(q) = 6000 + 10q$$

b) Write down the formula for the revenue function R(q).

$$R(q) = 25q$$

c) Please sketch the graph for C(q) and R(q) in the space provided below.



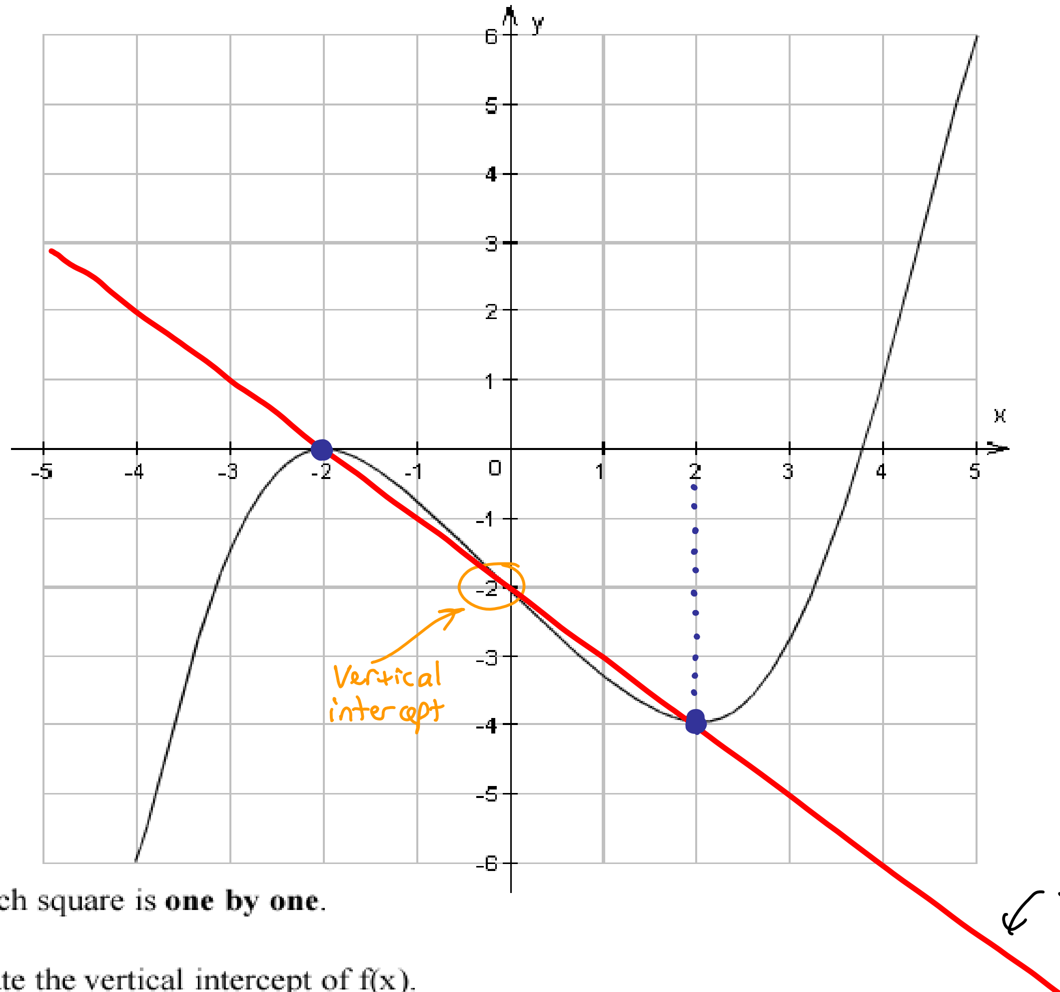
d) Find the break-even point.

$$C(q) = R(q)$$
$$6000 + 10q = 25q$$
$$6000 = 15q$$
$$400 = q$$

e) Estimate the profit generated by 800 units.

$$\pi(q) = R(q) - C(q), \quad \pi(800) = 25\cdot 800 - (6000 + 10\cdot 800)$$
$$= 6000$$

6. (3+4+3+4+3=17pts) Consider the following graph of the function **y = f(x)**.



Note: Each square is **one by one**.

slope = ave. r.o.c. in f(x) on [-2,2]

a) Estimate the vertical intercept of f(x).

intersection with y-axis

$y = -2$

b) Estimate the interval(s) on which f(x) is **decreasing**.

from $x=-2$ to $x=2$

interval notation: $[-2,2]$

c) Is the function concave up or down at x=2?

concave up

$x=2$

d) Find the **average rate of change** of f(x) from **x= −2** to **x= 2**.

$$\frac{\Delta y}{\Delta x} = \frac{f(2)-f(-2)}{2-(-2)} = \frac{-4-0}{4} = -1$$

e) Draw the secant line whose slope was used to find the answer to part d). shown on graph (red line)