**1.** (4+4 pts) Check if the following are power functions, if they are, write them in standard form $y = k \cdot x^p$, if not, explain why briefly.

**(a)** $y = 5\sqrt{x} =$ $5 \cdot x^{1/2}$

**(b)** $y = 5(.8)^x =$ not a power function, this is an exponential function

(for power functions when $x=0$, $y=0$ or undefined, hence not a power fn.)

**2.** (4+4 pts) Given $f(x) = x^2 - 3x$ and $g(x) = x + 5$, find the following:

**(a)** $f(g(x)) = f(x+5) = (x+5)^2 - 3(x+5)$

**(b)** $g(f(x)) = g(x^2-3x) = x^2 - 3x + 5$

**3.** (5+5 pts) The following table gives the distance of a runner running the Chicago marathon, measured $t$ minutes after the start of the race.

| t (mins) | 0 | 30 | 60 | 90 | 120 | 150 | 180 | 210 |
|---|---|---|---|---|---|---|---|---|
| d (miles) | 0 | 4.5 | 8.9 | 13 | 16.7 | 20.1 | 23.2 | 26.2 |

**(a)** Give your best estimate for the runner's speed at $120^{th}$ minute of the race. Give correct units. (Do not round your answer)

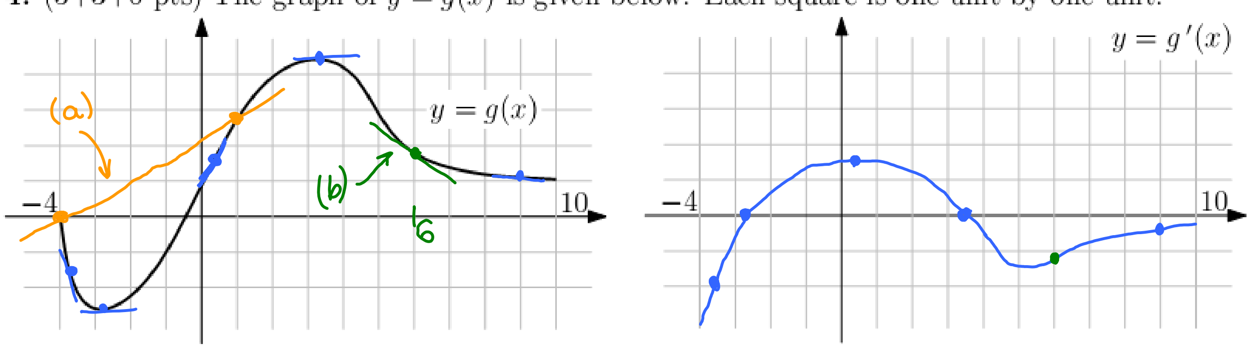
$$\text{speed} = d'(120) \approx \frac{d(150) - d(120)}{150 - 120} = .11\overline{3}\ \text{miles/minutes}$$

(Ave. r.o.c. on shortest interval is a good estimate for r.o.c.)

**(b)** Is the second derivative of the distance function positive or negative? Explain why.

negative, the velocity is decreasing (or: graph of $d(t)$ is concave down)

**4.** (3+3+6 pts) The graph of $y = g(x)$ is given below. Each square is one unit by one unit.




**(a)** Draw a line on the first graph that represents average rate of change in $g$ on the interval $[-4, 1]$.

**(b)** Draw a line on the first graph that represents $g'(6)$.

**(c)** Sketch the graph of $g'(x)$ on the second coordinate system.

**5.** (10 pts) Given $r(x) = \ln(2x+1)$, using smaller and smaller intervals, estimate $r'(3)$ accurate up to 2 decimals. Show at least 3 estimates.

| $h$ | difference quotient $= (r(3+h) - r(3))/h$ |
|---|---|
| $.01$ | $.2853$ |
| $.0001$ | $.2857$ |
| $0$ | |
| $-.0001$ | $.2857$ |
| $-.01$ | $.2861$ |

limit $= .2857$

$$= \left(\ln(2(3+h)+1) - \ln(2\cdot 3+1)\right)/h$$
$$= \left(\ln(2h+7) - \ln(7)\right)/h$$

**6.** (8+5 pts) Given the function $f(x) = 3x^2 - 5x$,

**(a)** Find and simplify $\dfrac{f(x+h) - f(x)}{h}$ (Do not compute a limit here)

$$\text{difference quotient} = \frac{3(x+h)^2 - 5(x+h) - (3x^2 - 5x)}{h}$$

$$= \frac{3x^2 + 6xh + 3h^2 - 5x - 5h - 3x^2 + 5x}{h} = \frac{h(6x + 3h - 5)}{h}$$

$$= 6x + 3h - 5$$

**(b)** Using the limit definition of the derivative, find the formula for $f'(x)$.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} 6x + 3h - 5 = 6x - 5$$

**7.** (4+4+4 pts) The altitude $H$ (in feet) of a helicopter is given by $H = f(t)$ where $t$ is the time since take-off measured in minutes.

**(a)** If $f(15) = 2000$, give the units of 15 and 2000 and interpret this statement in terms of the helicopter.

15 : minutes

2000 : feet

15 minutes after take-off the helicopter was at 2000ft altitude.

**(b)** If $f'(15) = -40$, give the units of 15 and $-40$ and interpret this statement in terms of the helicopter.

15 : minutes

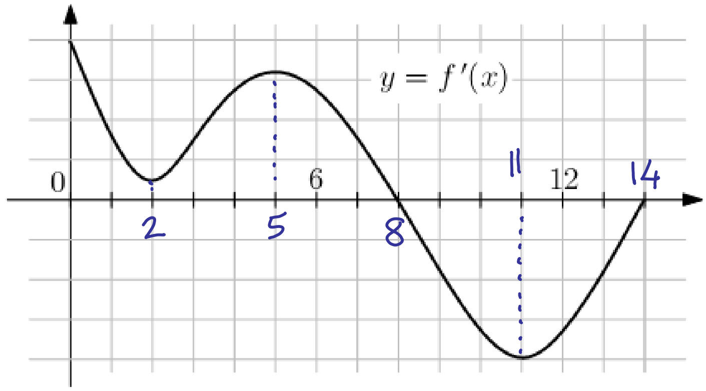
$-40$ : feet/min

Around 15 minutes after take-off, the helicopter's altitude was decreasing by 40 feet every minute.

**(c)** Using part (a) and part (b) estimate the altitude of the helicopter at $t = 18$.

$$f(18) = f(15) + 3\cdot(-40) = 2000 - 120 = 1880 \text{ feet}$$

**8.** (3+4+4 pts) $f(x)$ is the temperature measured $x$ hours since 12:00 noon. If the following graph gives the rate of change in $f(x)$, answer the following. ( This is not the graph of $y = f(x)$ )




**(a)** During which hours is the temperature decreasing?

8pm till 14pm $\quad$ $f'(x) < 0$, so $f(x)$ is decreasing

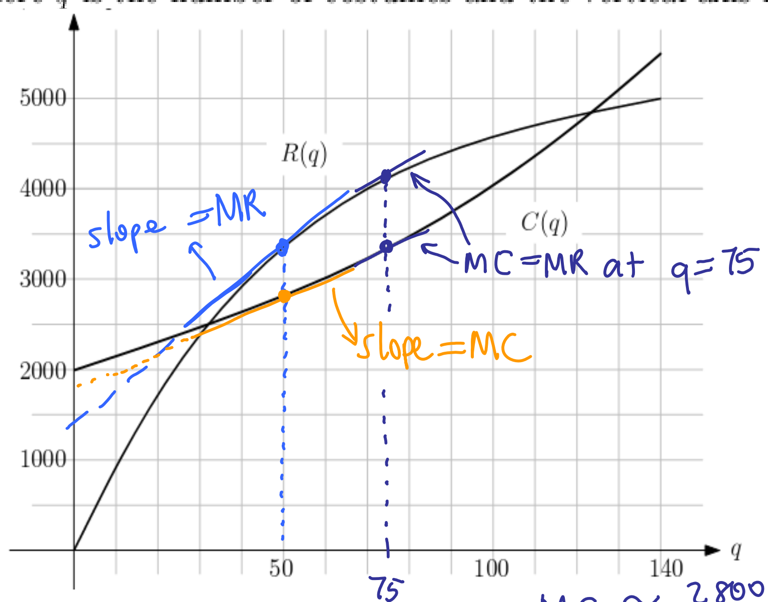
**(b)** During which hours is the graph of $f'$ concave down?

about 3:30pm till 8pm $\quad$ graph of $f'$ looks like ∩

**(c)** During which hours is the graph of $f$ concave down?

noon — 2pm and 5pm – 11pm $\quad$ $f'$ is decreasing, so $f'' < 0$

**9.** (4+4+4+4+4 pts) The total cost and revenue for a company producing halloween costumes is shown below, where $q$ is the number of costumes and the vertical axis is in dollars.




**(a)** Estimate the marginal cost for $q = 50$. Give correct units. $\quad MC \approx \dfrac{2800 - 1800}{50 - 0} = 20$ \$/costume

**(b)** Estimate the marginal revenue for $q = 50$. Give correct units. $\quad MR \approx \dfrac{3400 - 1400}{50 - 0} = 40$ \$/costume

**(c)** If the company has produced 50 costumes, should they increase production? Explain why in terms of marginal cost and marginal revenue.

Increase production because MR > MC, so profit would increase

**(d)** What value of $q$ maximizes profit? Include appropriate tangent lines on the graph above.

about $q = 75$ $\quad$ ( it seems $MC(75) = MR(75)$ )