Math 124 Sec 16 EXAM 3 SOLUTIONS FALL '07

1. (3+2=5pts) Suppose $F(x) = \int_0^x t^3 2^{t+2}\, dt$ when $x \geq 0$. Does the value of F increase or decrease as x increases? F increases because $F'(x) > 0$ for $x > 0$

   a. Find F'(x)

   using 2^{nd} FTC, $F'(x) = x^3 2^{x+2}$

   b. Find F(0)

   $F(0) = \int_0^0 t^3 2^{t+2}\, dt = 0$ (no area to shade)

2. (4+4+6+6 pts) Find the derivatives of the following functions. DO **NOT** SIMPLIFY YOUR ANSWER.

   a. $f(x) = 2x^3 - 1.25^x$

   $f'(x) = 6x^2 - 1.25^x \cdot \ln(1.25)$

   b. $k(t) = 3e^{t^2-4} - 11$

   $u = t^2 - 4$
   $u' = 2t$

   $f(u) = 3e^u - 11$
   $f'(u) = 3 \cdot e^u \cdot \underbrace{\ln(e)}_{=1}$

   $k'(t) = \underbrace{f'(u)}_{\frac{df}{du}} \cdot \underbrace{u'}_{\frac{du}{dt}} = 3e^{t^2-4} \cdot 2t$

   c. $h(x) = 4\ln(x^3 - 5x + 1)$

   $u = x^3 - 5x + 1$
   $u' = 3x^2 - 5$

   $f(u) = 4\ln(u)$
   $f'(u) = 4 \cdot \frac{1}{u}$

   $h'(x) = 4 \cdot \frac{1}{x^3 - 5x + 1} \cdot (3x^2 - 5)$

   d. $P(w) = 20\sqrt{2w^4 + 5}$

   $u = 2w^4 + 5$
   $\frac{du}{dw} = 2 \cdot 4w^3 + 0$

   $f(u) = 20 \cdot u^{1/2}$
   $\frac{df}{du} = 20 \cdot \frac{1}{2} \cdot u^{-1/2}$

   $\frac{dP}{dw} = \frac{df}{du} \cdot \frac{du}{dw} = 20 \cdot \frac{1}{2} \cdot (2w^4 + 5)^{-1/2} \cdot 2 \cdot 4w^3$

3. (13 pts.) Please find the equation of the line that is tangent to the graph of

$f(x) = 2x + \frac{3}{x}$ when x = 4. $\rightarrow$ $f(x) = 2x + 3 \cdot x^{-1}$

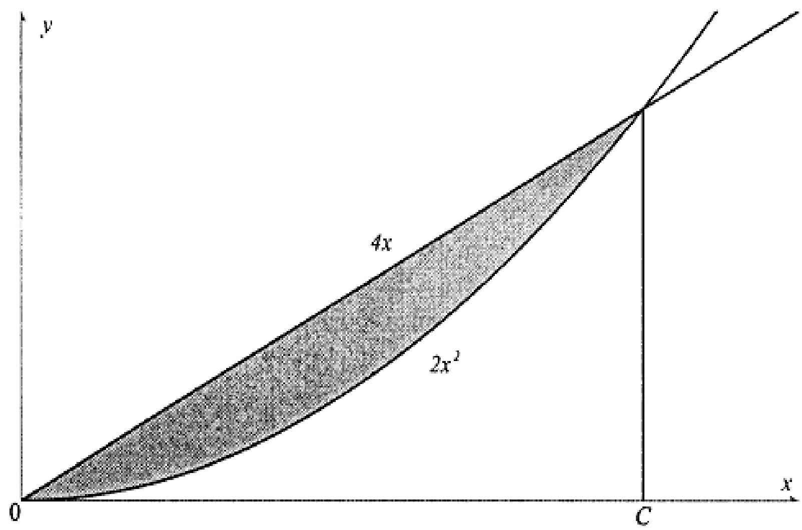
point of tangency: $(4, f(4)) = (4, 8.75)$

$f'(x) = 2 + 3 \cdot (-1) \cdot x^{-2}$

slope $= m = f'(4) = 2 + 3 \cdot (-1) \cdot \frac{1}{16} = 1.8125$

equation of tangent line: $y - 8.75 = 1.8125(x-4)$

4. (5+5pts)Consider the two functions $2x^2$ and $4x$ graphed below.



a. Find the intersection point of these two functions when x > 0. (ie. Find c above.)

algebraically: $2x^2 = 4x$

$2x^2 - 4x = 0$

$2x(x-2) = 0 \longrightarrow x=0$ or $x=2$

$c = 2$

in calculator:
$Y_1 = 4X$
$Y_2 = 2*X\wedge 2$
2^{nd} Trace $\rightarrow$ 5 : intersect

b. Write a single integral for the shaded area and estimate it correctly to 2 decimal places.

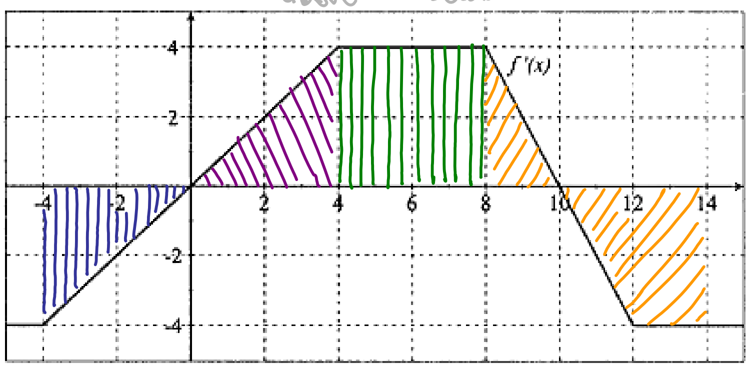
$$\int_0^2 \underbrace{(4x)}_{\text{above}} - \underbrace{(2x^2)}_{\text{below}}\, dx = 2.667$$

fnInt(4X − 2X^2 , X, 0, 2)

5. (3+3+3+3pts) Consider the graph below of $f'(x)$ (*The graph is NOT the graph of f(x).)*

Fill in the following table for the values of f(x) (***NOT f'(x)).***

FTC: $\underbrace{\int_a^b f'(x)\,dx}_{= \text{area above} - \text{area below x-axis}} = \underbrace{f(b) - f(a)}_{\text{change in } f}$



x	f(x)
-4	8
0	0
4	8
8	24
14	16

$a=-4$
$b=0$ $\longrightarrow$ $\int_{-4}^{0} f'(x) = f(0) - f(-4)$

$-8 = f(0) - 8$ ← given in table

$0 = f(0)$

Similarly $\int_0^4 f'(x)\,dx = f(4) - f(0)$

$8 = f(4) - 0$

$8 = f(4)$

$\int_4^8 f'(x)dx = f(8) - f(4)$

$16 = f(8) - 8$

$24 = f(8)$

$\int_8^{14} f'(x)\,dx = f(14) - f(8)$

$-8 = f(14) - 24$

$16 = f(14)$

6. (5+5pts)Suppose f(x) is a function such that $\int_0^3 f(x)dx = 3.25$ also that $\int_3^6 f(x)dx = -1$

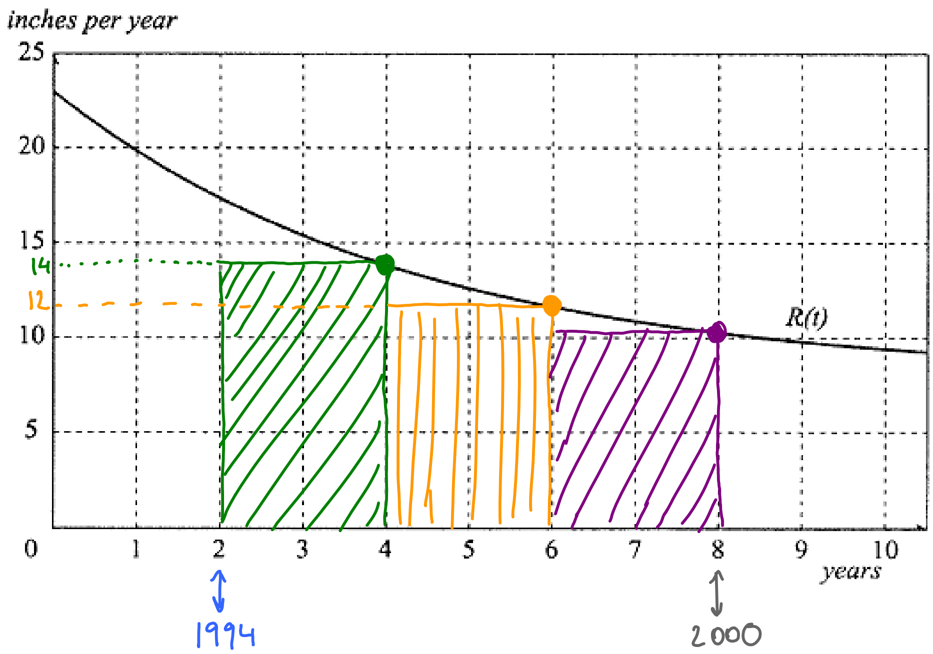
Let g(x) be a function such that $\int_0^3 g(x)dx = 1.4$ and $\int_3^6 g(x)dx = 2.2$. Find the following

integrals. (Note the limits of integration.)

a. $\int_3^6 \frac{f(x)}{2} + 4g(x)dx \quad = \frac{1}{2}\cdot\int_3^6 f(x)dx + 4\cdot\int_3^6 g(x)dx = \frac{1}{2}\cdot(-1) + 4\cdot(2.2) = 8.3$

b. $\int_0^6 3g(x)dx \quad = 3\cdot\left(\int_0^3 g(x)\,dx + \int_3^6 g(x)dx\right) = 3\cdot 1.4 + 3\cdot 2.2 = 10.8$

7. [2+5+3+5=15pts] The graph below shows $R(t)$, the rate at which a tree grows in inches per year, where t is given in years since 1992.



a. Write the definite integral that represents the total change in height for the tree between the years 1994 and 2000. Indicate what the units for the definite integral are.

Δheight $= \int_2^8 \underbrace{R(t)}_{\text{rate of change}}\,dt$ units: inches

b. Using a Right–Hand Sum with exactly three subintervals, estimate the definite integral you wrote in part *a.*

$$RHS = 2 \cdot 14 + 2 \cdot 12 + 2 \cdot 10.5 = 73$$

c. *Carefully* shade in the region whose area is given by the right-hand sum.

see the graph above

d. If the tree's height was 42 inches in the year 1994, use your estimate in part *b* to find the tree's total height in the year 2000.

initial height (in 1994) was 42 inches

Change in height $= \int_2^8 R(t)\,dt \approx 73$

final height (in 2000) $= 42 + 73 = 115$ inches

8. (10+5=15pts)Samantha has a small business that makes the most huggable teddy bears on the market. The total cost to produce q $Super-Soft^R$ Teddy Bears is given by C(q).

Fixed Costs are $3,000 and $C'(q)$ is given by the table below.

q	C'(q) ($)
0	21
5	23
10	26
15	31
20	36
25	43
30	51

needed in part a

needed in part b

a. Estimate the total cost to produce 20 $Super-Soft^R$ teddy bears.

$$C(20) = C(0) + \int_0^{20} C'(q)\,dq$$

$$\approx 3000 + \frac{505+580}{2} = \$3542.50$$

$$LHS = 5\cdot 21 + 5\cdot 23 + 5\cdot 26 + 5\cdot 31 = 505$$

$$RHS = 5\cdot 23 + 5\cdot 26 + 5\cdot 31 + 5\cdot 36 = 580$$

b. Estimate the cost of producing 10 more bears.

$$\underbrace{C(30) - C(20)}_{\text{additional cost}} = \int_{20}^{30} C'(q)\,dq$$

$$\approx \frac{395+470}{2} = \$432.50$$

$$LHS = 5\cdot 36 + 5\cdot 43 = 395$$

$$RHS = 5\cdot 43 + 5\cdot 51 = 470$$