

Show your work in all problems.

1. Find the derivatives of the following functions:

(a) $k(x) = \underbrace{x^5}_f \cdot \underbrace{3^x}_g$

product rule: $(f \cdot g)' = f' \cdot g + f \cdot g'$

$$k'(x) = \underbrace{5x^4}_{f'} \cdot \underbrace{3^x}_g + \underbrace{x^5}_f \cdot \underbrace{3^x \cdot \ln(3)}_{g'}$$

(b) $s(t) = \frac{\underbrace{4-t^2}_f}{\underbrace{5t+1}_g}$

quotient rule: $\left(\frac{f}{g}\right)' = \frac{f' \cdot g - f \cdot g'}{(g)^2}$

$$s'(t) = \frac{-2t \cdot (5t+1) - (4-t^2) \cdot 5}{(5t+1)^2}$$

2. A company producing snowshoes wants to sell them at unit price p dollars. The quantity sold q depends on p , so we write $q = f(p)$. If we know $f(150) = 5000$ and $f'(150) = -25$, answer the following:

(a) Find the revenue when $p = 150$. (Revenue = selling price \times quantity)

$$R(p) = p \cdot q = p \cdot f(p)$$

$$R(150) = 150 \cdot f(150) = 150 \cdot 5000 = \$750000$$

(b) Find $\left(\frac{dR}{dp}\right)$ at $p = 150$.
r.o.c. in revenue as p changes

$$R(p) = p \cdot f(p)$$

$$R'(p) = 1 \cdot f(p) + p \cdot f'(p)$$

$$R'(150) = 1 \cdot 5000 + 150 \cdot (-25)$$

$$= 5000 - 3750 = 1250 \text{ } \cancel{\$/\cancel{\$}}$$

units of R' : $\frac{\text{units of } R}{\text{units of } p} = \frac{\$}{\$}$

derivative of p is 1:

$$\frac{d}{dp}(p) = 1 \cdot p^0 = 1$$

(just like $y=x$ has derivative $y'=1$)

Similar to #35, 38 in 3.4

(c) Should they increase the selling price or decrease it to make more profit? Explain why using your answer to part (b).

Since $R'(150) > 0$, it means R increases when p is near 150.

Hence the company should increase selling price to make more profit.