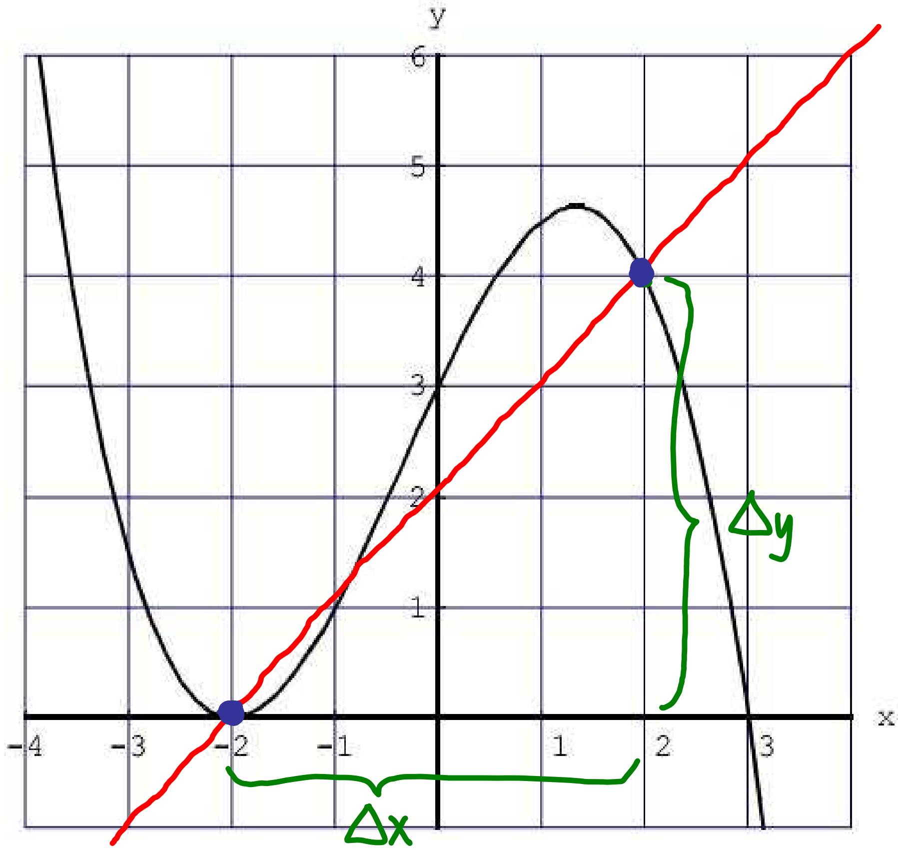
1. (4+2+3+6+2=17 pts) Consider the following graph of $f(x)$:



a) Estimate the average rate of change of $f(x)$ between $x=-2$ and $x=2$.

Ave r.o.c. $= \dfrac{\Delta y}{\Delta x} = \dfrac{f(2) - f(-2)}{2-(-2)} = \dfrac{4-0}{4} = 1$

b) To the given graph above, add the line whose slope you used to find the answer to part (a). It's the secant line drawn on the graph (red)

c) Estimate the interval(s) on which $f(x)$ is increasing.

From $x=-2$ till $x=1.3$

interval notation: $[-2, 1.3]$

d) Estimate the interval(s) on which $f(x)$ is decreasing.

From $x=-3.8$ to $x=-2$ and from 1.3 to 3.2

interval notation: $[-3.8, -2]$ and $[1.3, 3.2]$

e) Is $f(x)$ concave up or down at $x=2$?

it is concave down: 2, f(2) Graph bends down

2. (5+6+6=17 pts) Sales of music compact disks (CDs) increased rapidly throughout the 1990s. Sales were **482** million CDs in **1993** and **938** million CDs in **1999**.

a) Find a linear formula for CD sales, S, as a function of the number of the years, t, since **1991** (Where $t = 0$ corresponds to **1991**.)

year	t	S
1991 ↔	0	?
1993 ↔	2	482
1999 ↔	8	938

$$S = b + mt$$

$$m = \frac{\Delta S}{\Delta t} = \frac{938-482}{8-2} = \frac{456}{6} = 76$$

$$S = b + 76t, \quad t=2, \ S=482$$

$$482 = b + 76 \cdot 2$$

$$330 = b$$

$$\boxed{S = 330 + 76t}$$

b) What is the slope of the line found in part (a)? Give units and **interpret** your answer.

Slope = 76 millions of CDs per year

interpretation: between 1993 and 1999 each year sales in CDs increased by about 76 million CDs.

c) What is the vertical intercept of the line found in part (a)? Give units with your answer and **interpret** it.

vertical intercept = b = 330 million CDs

interpretation: in 1991 330 million CDs were sold

3. (5+3+5+2+3=18 pts) A company producing lamps has the fixed cost of 2000 dollars and the variable cost of 12.50 dollars per lamp. The company sells lamps at a price of 37.50 dollars each.

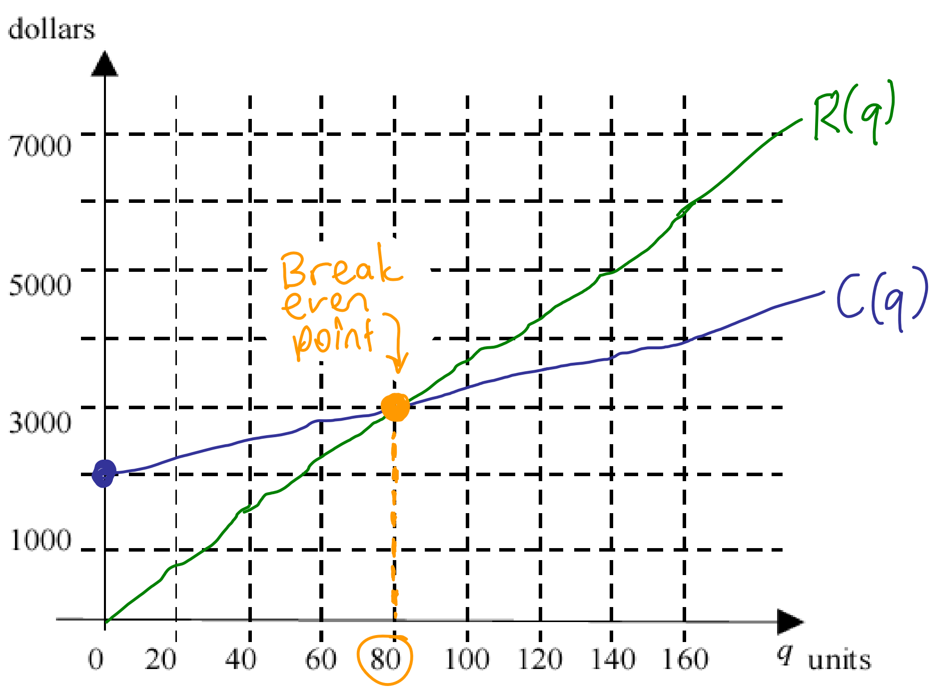
a) Write the formula for the cost function:

$$C(q) = 2000 + 12.50q$$

b) Write the formula for the revenue function:

$$R(q) = 37.50q$$

c) Sketch the cost and revenue functions in the coordinate system below:



d) Estimate the break-even point, and label it on the graph.

$$C(q) = R(q)$$

$$2000 + 12.50q = 37.50q$$

$$2000 = 25q \longrightarrow q = \frac{2000}{25} = 80$$

e) What is the profit when they sell 160 lamps?

$$\pi(q) = R(q) - C(q)$$

$$\pi(160) = 37.50 \cdot 160 - (2000 + 12.50 \cdot 160) = 2000$$

4. (6+5+5 =16 pts) Three functions are given as below.

(1)

	x	f(x)	
+4	-3	27	−8
+4	1	19	−8
+4	5	11	−8
	9	3	

(2)

	t	g(t)	
+2	–2	1530.9	$\cdot .11111$
+2	0	170.1	$\cdot .11111$
+2	2	18.9	$\cdot .11111$
	4	2.1	

(3)

	u	h(u)	
+3	0	2	+8
+3	3	10	+6
+3	6	16	+16
	9	32	

a) Identify each of the functions as either linear, exponential or neither.

(1) Linear (ave r.o.c. $= \frac{-8}{4} = -2$)

(2) exponential ($a^2 = \cdot .11111$, $a = .3333$)

(3) neither (not linear, not exponential)

slope not constant inconsistent growth factor

b) Find a formula for the linear function.

$$y = b + mx \qquad m = \frac{\Delta y}{\Delta x} = \frac{-8}{4} = -2$$

$$y = b + (-2)x$$

when $x=1$, $y=19$

$$19 = b + (-2)\cdot 1 \longrightarrow b = 21$$

$$y = 21 - 2x$$

c) Find a formula for the exponential function.

$$g(t) = P_0 \cdot a^t$$

$$\frac{18.9}{170.1} = \frac{g(2)}{g(0)} = \frac{P_0 \cdot a^2}{P_0 \cdot a^0} = a^2$$

$$\sqrt{\frac{18.9}{170.1}} = a$$

$$\shortparallel$$

$$.333... = .\overline{3}$$

$$g(t) = 170.1 \cdot (.\overline{3})^t$$

5. (5+5 = 10 pts) Suppose that you deposit \$7000 in an account with a 4.8% annual interest rate. Write the formula for the balance after *t* years, if

a) the interest is compounded annually:

$$P(t) = 7000 \cdot (1 + .048)^t$$

$$= 7000 \cdot (1.048)^t$$

b) the interest is compounded continuously:

$$P(t) = 7000 \cdot e^{.048t}$$

6. (7+7 = 14 pts) After 1 year, a certain radioactive isotope has lost 12% of its original content.

after one year 88% left

a) Find the **continuous** decay rate. $\to k = ?$

t	P
0	P_0
1	$P_0 \cdot \frac{88}{100}$

$\to$ $P(t) = P_0 \cdot (.88)^t = P_0 \cdot e^{kt}$

$$.88^t = e^{kt}$$

$$.88 = e^k$$

$$\ln(.88) = k$$

$$= -.12783$$

b) Find the half-life of this isotope.

solve $P(t) = \frac{1}{2} P_0$

$$P_0 \cdot e^{-.12783t} = \frac{1}{2} P_0$$

$$e^{-.12783t} = \frac{1}{2}$$

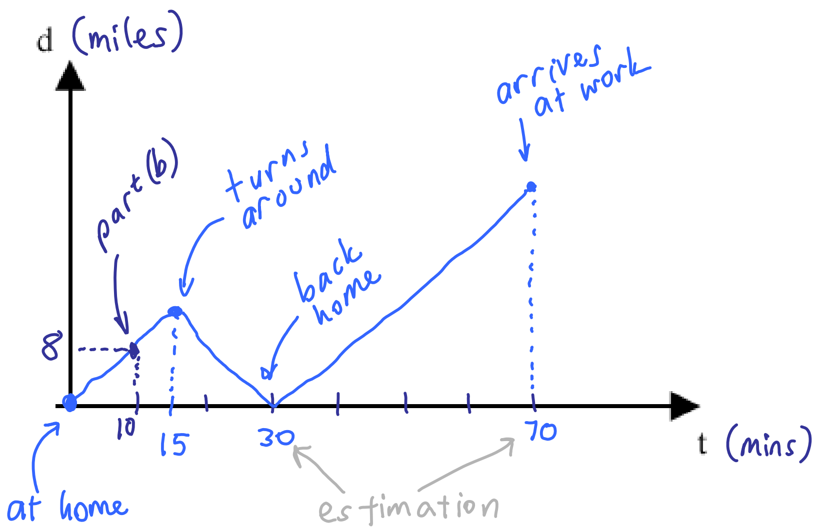
take ln of both sides

$$-.12783t = \ln\left(\tfrac{1}{2}\right)$$

$$t = \frac{\ln(1/2)}{-.12783} = 5.4224$$

7. (5+3 = 8 pts) A woman leaves her home going to her working place which is straight down the road 40 minutes away from her home. After 15 minutes, she realizes that she has forgotten her notebook at home, so she turns around and drives back home from that point. After taking her notebook, she again drives to work.

a) Sketch a possible graph of her distance d = f (t), in miles, from her home as a function of time, in minutes.



b) In the context of this problem, what does f (10) = 8 mean?

10 minutes after leaving home, she is 8 miles away from home.