

1. [4 points] The volume  $V$  of a sphere is proportional to the cube of its radius  $r$ . Given that the sphere with radius 1 has volume  $\frac{4\pi}{3}$ , write a formula for  $V$  in terms of  $r$ .

$$V = k \cdot r^3$$

$$\frac{4\pi}{3} = k \cdot 1 \longrightarrow k = \frac{4\pi}{3} \qquad V = \frac{4\pi}{3} \cdot r^3$$

2. [4 points] Determine whether  $\frac{3}{\sqrt{4x}}$  is a power function. If it is, express it as  $k \cdot x^p$ .

$$\frac{3}{\sqrt{4x}} = \frac{3}{2\sqrt{x}} = \frac{3}{2} \cdot \frac{1}{\sqrt{x}} = \frac{3}{2} \cdot x^{-1/2}$$

3. [4 points] Use the variable  $u$  for the inside function to express  $e^{(x^2-5)}$  as a composite function.

$$u(x) = x^2 - 5$$

$$f(x) = e^x \longrightarrow f(u) = f(x^2 - 5) = e^{(x^2 - 5)} \quad \checkmark$$

4. [4 + 3 + 3 points] Consider the function given in the following table.

$x$	0	5	10	15	20
$f(x)$	0	40	60	70	74

- (a) Approximate  $f'(10)$ .

$$f'(10) \approx \text{ave r.o.c. on } [10, 15] = \frac{70 - 60}{15 - 10} = \frac{10}{5} = 2$$

↓  
smallest interval containing 10

([5, 10] is also ok:  $\frac{60 - 40}{10 - 5} = 4$ )

- (b) Does the derivative of  $f(x)$  appear to be positive or negative over the given interval? Explain.

↓  
 $f'(x)$  appears to be positive

since values of  $f(x)$  are increasing. (we don't know intermediate values though...)

- (c) Does the second derivative of  $f(x)$  appear to be positive or negative over the given interval? Explain.

$f''(x)$  appears to be negative:  $f'(x)$  is decreasing

5. [4 + 4 + 4 + 4 **points**] A frozen turkey has just been removed from a freezer and placed on a kitchen table (at room temperature).  $P(t)$  measures the temperature of the turkey (in  $^{\circ}F$ )  $t$  hours after being removed from the freezer.

(a) Do you expect  $P'(t)$  to be positive or negative? Explain why.

positive because it will defrost ( $P(t)$  will increase)

(b) In the context of this question, explain the practical meaning of  $P(4) = 47$ . Provide units.

4 hours after being removed from freezer, the turkey was  $47^{\circ}F$ .

(c) In the context of this question, explain the practical meaning of  $P'(4) = 2$ . Provide units.

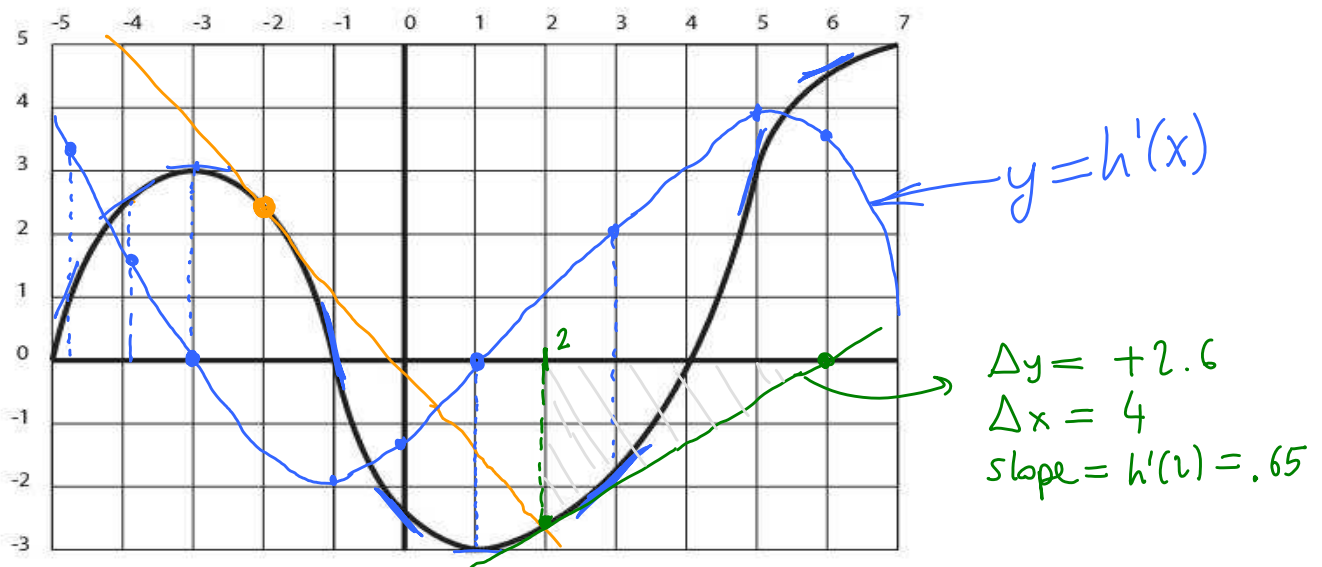
Near 4 hours after being removed from freezer, the turkey's temperature increases by about  $2^{\circ}F$  every additional hour.

(d) Using the facts that  $P(4) = 47$  and  $P'(4) = 2$ , estimate  $P(6)$ .

$$P(6) \approx P(4) + \Delta t \cdot P'(4)$$

$$= 47 + 2 \cdot 2 = 51$$

6. [10 + 4 + 4 **points**] The graph of a function  $h(x)$  is shown below.



(a) Sketch the graph of  $h'(x)$  in the same coordinate system as  $h(x)$ .

(b) Sketch a line whose slope is  $h'(-2)$  on the graph of  $h(x)$ .

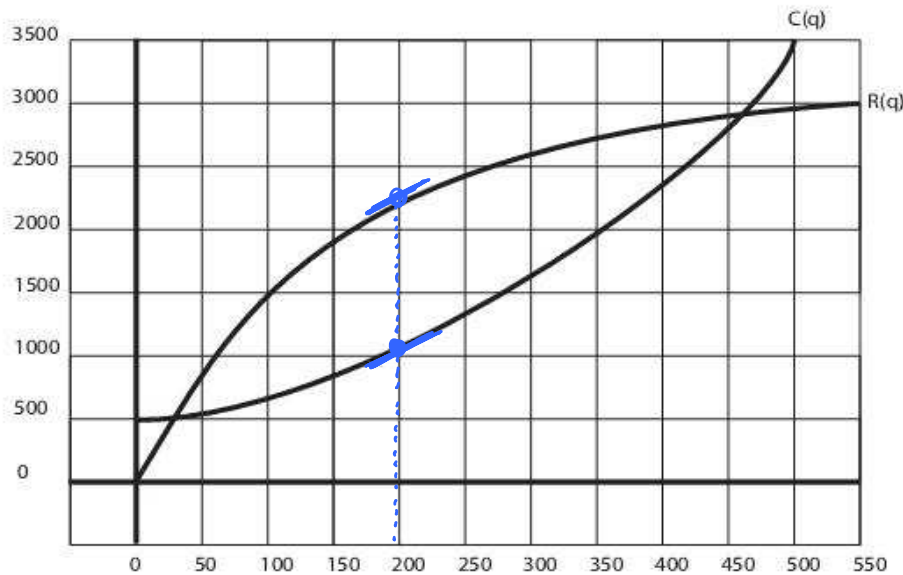
(c) Estimate  $h'(2)$ .  $\approx \frac{\Delta y}{\Delta x} = \frac{2.6}{4} = .65$

7. [13 points] Let  $f(x) = 5x - x^2 + 3$ . Using the limit definition of the derivative, compute  $f'(x)$ . No credit will be assigned for any other method.

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{5(x+h) - (x+h)^2 + 3 - (5x - x^2 + 3)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{5x + 5h - x^2 - 2xh - h^2 + 3 - 5x + x^2 - 3}{h} = \lim_{h \rightarrow 0} \frac{h(5 - 2x - h)}{h} = 5 - 2x$$

8. [4+4+8 points] A family has started up a company in order to mass produce their traditional hot sauce. They have determined that the cost and revenue functions for producing and selling  $q$  bottles of hot sauce are as in the graph below.



(a) How many bottles of hot sauce should they make in order to maximize profit? Explain why in terms of the marginal cost and revenue.

About 200 bottles maximize profit, because  $MR = MC$ , and later  $MC > MR$ , which means increasing production decreases revenue.

(b) Assuming that they have already produced 350 bottles, should they continue production? Explain why in terms of the marginal cost and revenue.

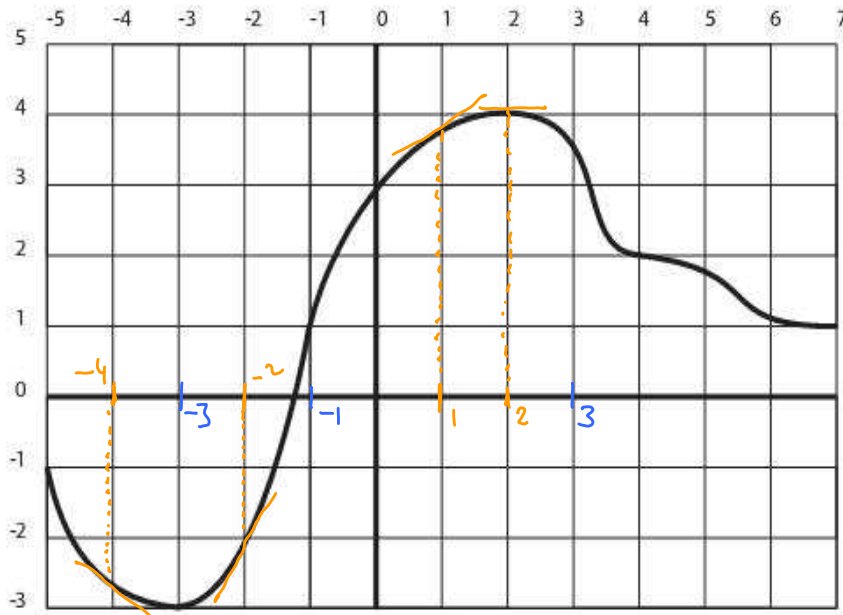
NO. Because  $MC(350) > MR(350)$ , so increasing production would decrease profit

(c) The cost function is  $C(q) = 500 \cdot (1.004)^q$ . Estimate the marginal cost at  $q = 100$ . Make sure that your estimate is correct up to one decimal place. Show at least three estimates.

$h$	difference quotient $(C(100+h) - C(100))/h$
$\downarrow$ .01	2.97538
.0001	2.97532
$\uparrow$ -.0001	2.97532
-.01	2.97526

estimate:  $C'(100) \approx 2.97532$

9. [9 + 6 points] Below is a graph of  $Q(t)$ .



(a) In the following table, enter whether the following functions are positive, negative, or 0.

$t$	$Q(t)$	$Q'(t)$	$Q''(t)$
-3	-	0	+
-1	+	+	0
3	+	-	-

function values

slope of tangent line

concavity

inflection point, concavity changes

(b) Order the following numbers from least to greatest:  $Q'(-4)$ ,  $Q'(-2)$ ,  $Q'(1)$ ,  $Q'(2)$ .

- + + 0

$$Q'(-4) < Q'(2) < Q'(1) < Q'(-2)$$