1. (4+4+4+4=16 pts)Compute the derivates of the given functions. SHOW YOUR WORK.

a) $y = 3x^4 + 5x + 2$

$y' = 3 \cdot 4x^3 + 5$

b) $C(q) = 3\ln(5^q + q^5)$

$u = 5^q + q^5$

$u' = 5^q \cdot \ln(5) + 5q^4$

$f(u) = 3\ln(u)$

$f'(u) = 3 \cdot \frac{1}{u}$

$C'(q) = f'(u) \cdot u'$

$= 3 \cdot \frac{1}{(5^q + q^5)} \cdot (5^q \cdot \ln(5) + 5 \cdot q^4)$

c) $f(t) = 2e^{t^2 - 5t}$

$w = t^2 - 5t$

$w' = 2t - 5$

$g(w) = 2e^w$

$g'(w) = 2e^w \cdot \underbrace{\ln(e)}_{=1}$

$\frac{df}{dt} = \frac{dg}{dw} \cdot \frac{dw}{dt} = g'(w) \cdot w'$

$= 2e^{t^2 - 5t} \cdot (2t - 5)$

d) $w = (3x + \frac{2}{x})^{17}$

$u = 3x + \frac{2}{x}$

$= 3x + 2 \cdot x^{-1}$

$u' = 3 + 2 \cdot (-1) \cdot x^{-2}$

$w = f(u) = u^{17}$

$f'(u) = 17u^{16}$

$\frac{dw}{dx} = f'(u) \cdot u'$

$= 17(3x + \frac{2}{x})^{16} \cdot (3 - \frac{2}{x^2})$

2. (10 pts) Using the graph of f'(x) given below, fill in the table.

FTC

$$\int_2^4 f'(x)\,dx = f(4) - f(2)$$

$$2 = 2 - f(2)$$

$$0 = f(2)$$

$$\int_4^6 f'(x)\,dx = f(6) - f(4)$$

$$-2 = f(6) - 2$$

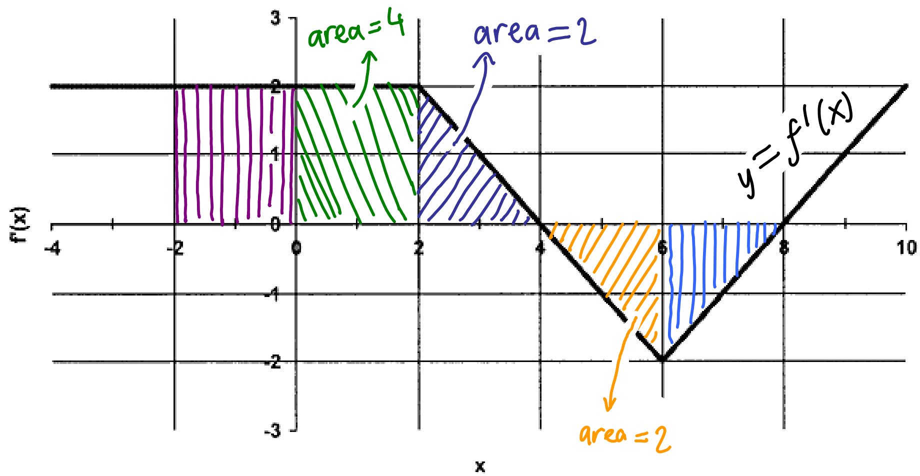
$$0 = f(6)$$

$$\int_0^2 f'(x)\,dx = f(2) - f(0)$$

$$4 = 0 - f(0)$$

$$-4 = f(0)$$

x	-2	0	2	4	6	8
f(x)	−8	−4	0	2	0	−2



$$\int_{-2}^0 f'(x)\,dx = f(0) - f(-2)$$

$$4 = -4 - f(-2) \longrightarrow f(-2) = -8$$

$$\int_6^8 f'(x)\,dx = f(8) - f(6)$$

$$-2 = f(8) - 0$$

3. (10+4=14 pts) $f(x) = \sqrt{3x+6}$

a) Find the equation of the tangent line at $x = 1$.

$$f(x) = (3x+6)^{1/2}$$

$$f'(x) = \tfrac{1}{2}(3x+6)^{-1/2} \cdot 3$$

$$m = \text{slope} = f'(1) = \tfrac{1}{2} \cdot \tfrac{1}{\sqrt{9}} \cdot 3 = \tfrac{1}{2}$$

point of tangency:

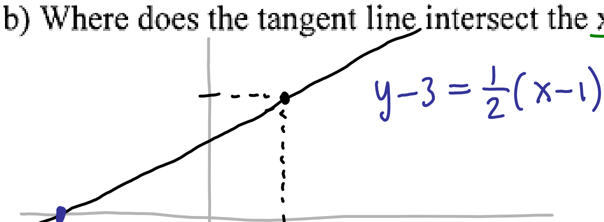
$$(1, f(1)) = (1, \sqrt{9}) = (1, 3)$$

equation: $y - 3 = \frac{1}{2}(x - 1)$

or: $y = \frac{1}{2}x + \frac{5}{2}$

b) Where does the tangent line intersect the x-axis?

consists of points with $y=0$



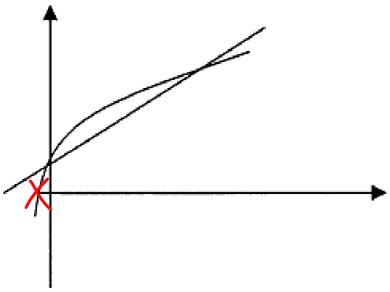
set $y=0$, solve for x

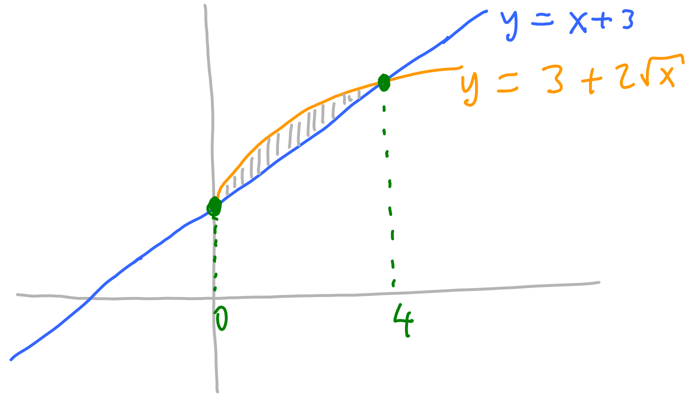
$$0 - 3 = \tfrac{1}{2}(x-1)$$

$$-6 = (x-1)$$

$$-5 = x$$

4. (16pts) $f(x)=x+3$ and $k(x)=2\sqrt{x}+3$.





a). Find the x-coordinates of the points of intersection of f and k.

$x=0$ and $x=4$

algebraically:

$$x+3 = 3+2\sqrt{x}$$
$$x = 2\sqrt{x}$$
square both sides
$$x^2 = 4x$$
$$x^2-4x=0 \rightarrow x(x-4)=0$$
$x=0$ or $x=4$

b). Write a **single** integral for the area of the region bounded by the two curve f and k, including appropriate limits of integration.

$$\text{area} = \int_0^4 \underbrace{(3+2\sqrt{x})}_{\text{above}} - \underbrace{(x+3)}_{\text{below}}\,dx$$

c). Estimate the value of the integral in part a) to at least two decimal places.

$$\text{fnInt}((3+2\sqrt{(x)})-(X+3)), X, 0, 4) = 2.667$$

5. (8 pts) Given $\int_0^1 f(x)dx=-7$, $\int_0^1 g(x)dx=10$, $\int_0^5 f(x)dx=3$. What is the value of:

see Ch5 FOT

a). $\int_0^1 (4f(x)+9g(x))dx = 4\int_0^1 f(x)\,dx + 9\int_0^1 g(x)\,dx$

$$= 4\cdot(-7)+9\cdot 10 = 62$$

b) $\int_1^5 f(x)dx$

$$\underbrace{\int_0^1 f(x)\,dx}_{-7} + \int_1^5 f(x)dx = \underbrace{\int_0^5 f(x)dx}_{3}$$

add 7 to both sides

hence $\int_1^5 f(x)\,dx = 3+7 = 10$

6. (24 pts) The table below gives the marginal cost of producing q radios. Assume that $C(0) = 2007$ dollars.

Q	0	100	200	300	400
MC(q) $= C'(q)$	60	52	46	42	40

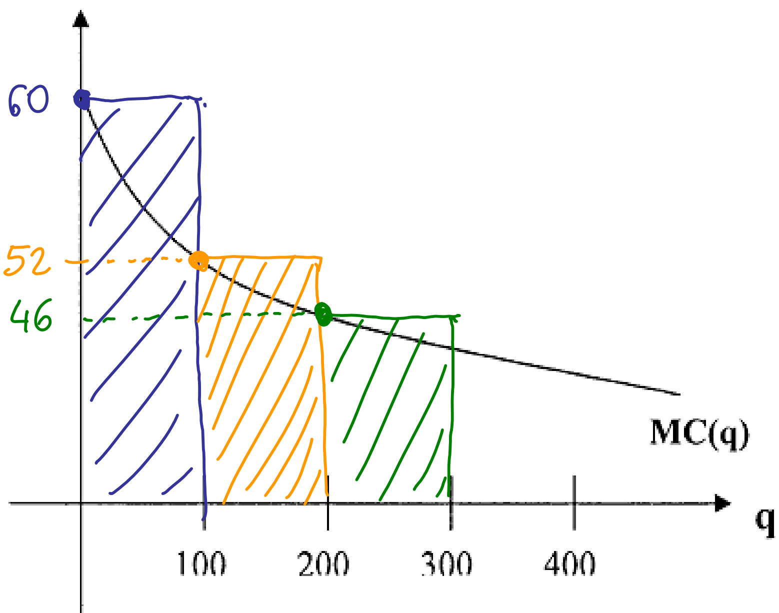
a) Give left hand sum and right hand sum estimate for the total variable cost ~~$C(q)$~~ of producing 300 units.

$$LHS = 100 \cdot (60 + 52 + 46) = 15800$$
$$RHS = 100 \cdot (52 + 46 + 42) = 14000$$

estimate for total variable cost.

b) Below, please draw the rectangles corresponding to the left hand sum estimate in part (a).

at each small interval choose the height as the function value at left endpoint

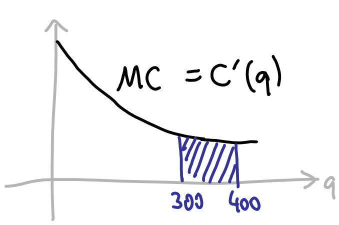


~~b)~~ c). Give your best estimate for the total cost of manufacturing 300 radios.

$$\Delta C \approx \frac{LHS + RHS}{2} = \frac{15800 + 14000}{2} = 14900$$
$$C(300) = C(0) + \Delta C = 2007 + 14900 = \$16907$$

~~c)~~ d). Explain the meaning of $\int_{300}^{400} MC(q)dq$ in the context of this problem. Include correct units!

additional cost for producing 100 more radios after 300 have been prodrced, measured in \$.



$$\int_{300}^{400} C'(q)dq = C(400) - C(300)$$

7. (12 pts) $F(b) = \int_0^b 3^x \, dx$

a). What is $F(0)$?

$F(0) = \int_0^0 3^x \, dx = 0$ (no area to shade)

b). Does the value of F increase or decrease as b increases? (Assume $b \geq 0$).

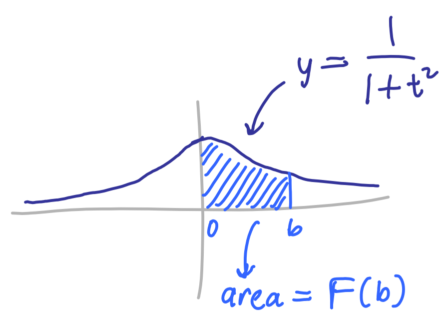
according to part (c), $F'(b) = 3^b > 0$
hence F is increasing

c). What is $F'(b)$?

by 2^{nd} FTC, $F'(b) = 3^b$

Extra example:
(not in exam)

$$F(b) = \int_0^b \frac{1}{1+t^2} \, dt$$



is $F(b)$ increasing?

$F'(b) = \frac{1}{\underbrace{1+b^2}_{\geq 1}} > 0$, hence F is increasing