

Show your work in all problems.

1. Find the derivatives of the following functions:

(a)  $k(x) = \underbrace{x^3}_{f'} \cdot \underbrace{5^x}_g$

product rule:  $(f \cdot g)' = f' \cdot g + f \cdot g'$

$$k'(x) = \underbrace{3x^2}_{f'} \cdot \underbrace{5^x}_g + \underbrace{x^3}_f \cdot \underbrace{5^x \ln(5)}_{g'}$$

(b)  $s(t) = \frac{\underbrace{t^2 + 1}_f}{\underbrace{3t - 1}_g}$

quotient rule:  $\left(\frac{f}{g}\right)' = \frac{f' \cdot g - f \cdot g'}{(g)^2}$

$$s'(t) = \frac{2t \cdot (3t - 1) - (t^2 + 1) \cdot 3}{(3t - 1)^2}$$

2. A company producing snowshoes wants to sell them at unit price  $p$  dollars. The quantity sold  $q$  depends on  $p$ , so we write  $q = f(p)$ . If we know  $f(200) = 5000$  and  $f'(200) = -30$ , answer the following:

(a) Find the revenue when  $p = 200$ . (Revenue = selling price  $\times$  quantity)

$$R(p) = p \cdot q = p \cdot f(p)$$

$$R(200) = 200 \cdot f(200) = 200 \cdot 5000 = \$1000000$$

(b) Find  $\frac{dR}{dp}$  at  $p = 200$ .

units of  $R'$ :  $\frac{\text{units of } R}{\text{units of } p} = \frac{\$}{\$}$

derivative of  $p$  is 1:

$$\frac{d}{dp}(p) = 1 \cdot p^0 = 1$$

(just like  $y=x$  has derivative  $y'=1$ )

$$R(p) = \underbrace{p}_{\text{r.o.c. in revenue}} \cdot \underbrace{f(p)}_{\text{as } p \text{ changes}}$$

$$R'(p) = 1 \cdot f(p) + p \cdot f'(p)$$

$$R'(200) = 1 \cdot f(200) + 200 \cdot f'(200) = 1 \cdot 5000 + 200 \cdot (-30)$$

$$= 5000 - 6000 = -1000 \frac{\$}{\$}$$

(c) Should they increase the selling price or decrease it to make more profit? Explain why using your answer to part (b).

Since  $R'(200) < 0$  it means  $R$  decreases when  $p$  is near 200. Hence the company shouldn't increase the selling price.

Similar to #35, 38 in 3.4