

Show your work in all problems.

1. Find the doubling time for the balance in a bank account for which interest is compounded continuously at continuous rate of 3%.

t	balance
0	P_0
?	$2P_0$

$$2P_0 = P_0 \cdot e^{.03t} \quad \leftarrow \text{solve for } t$$

$$2 = e^{.03t}$$

take logarithm of both sides

$$\ln 2 = .03t \cdot \underbrace{\ln(e)}_1 \rightarrow t = \frac{\ln(2)}{.03} = 23.10491$$

2. Using smaller and smaller intervals estimate the derivative of $f(x) = \sin(x^2)$ at $x = 2$.

$$f'(2) = \lim_{h \rightarrow 0} \frac{f(2+h) - f(2)}{h}$$

difference quotient

$$= \lim_{h \rightarrow 0} \frac{\sin((2+h)^2) - \sin(2^2)}{h}$$

h	difference quotient
.01	-2.5595
.0001	-2.61403
0	undefined (division by zero)
-.0001	-2.61511
-.01	-2.6675

limit
= -2.6145

3. State the Fundamental Theorem of Calculus. Using FTC, find the variable cost for producing 10 more items after 30th item if the marginal cost is given by $C'(q) = 40 + \sqrt{q}$

FTC: if $F'(t)$ is continuous,

$$\int_a^b F'(t) dt = F(b) - F(a)$$

integral of rate of change gives total change

$$\text{additional cost} = \underbrace{C(40) - C(30)}_{\Delta C} \stackrel{\text{FTC}}{=} \int_{30}^{40} C'(q) dq = \text{fnInt}(40 + \sqrt{x}, x, 30, 40) = 459.11$$

4. Using shortcuts, find the derivative of $g(x) = \underbrace{(5x)}_u \cdot \underbrace{\ln(x^2 + 5x)}_v$.

$$g'(x) = u' \cdot v + u \cdot v' \quad (\text{product rule})$$

$$= 5 \cdot \ln(x^2 + 5x) + 5x \cdot \frac{1}{x^2 + 5x} \cdot (2x + 5)$$

chain rule

$$\frac{d}{dx}(\ln w) = \frac{1}{w} \cdot w'$$

5. Make a table of signs for the first and second derivatives of $r(x) = x^4 - 4x^3 + 5$. List local minima, local maxima and inflection points.

$$r'(x) = 4x^3 - 12x^2 = 4x^2(x - 3)$$

$$r'(x) = 0 \rightarrow x^2 = 0 \text{ or } x - 3 = 0$$

$$r''(x) = 12x^2 - 24x = 12x(x - 2)$$

$$r''(x) = 0 \rightarrow x = 0 \text{ or } x - 2 = 0$$

x	0	2	3
sign of r'	-	0	-
sign of r''	+	0	-

local min: $x=3$
local max: none

inflection points: $x=0, 2$