

Show your work in all problems.

1. Find the formula of the function $y = g(x)$ whose graph is obtained by first shifting $f(x) = x^3 - 2x$ to the right by 1 unit and then stretching it vertically by a factor of 5.

$$x^3 - 2x \xrightarrow{\text{shift right}} (x-1)^3 - 2(x-1) \xrightarrow{\text{stretch vertically}} 5((x-1)^3 - 2(x-1))$$

2. Write the following power functions in standard form: $y = k \cdot x^p$

(a) $y = (3x)^4 = 3^4 \cdot x^4 = 81 \cdot x^4$, $k=81$, $p=4$

(b) $y = \frac{1}{5x^3} = \frac{1}{5} \cdot \frac{1}{x^3} = \frac{1}{5} \cdot x^{-3}$, $k=.2$, $p=-3$

3. Given $f(x) = 5^x$, estimate $f'(2)$ accurate up to 3 decimals (using at least 2 different intervals from the left and from the right).

$f'(2) \approx$ ave. r.o.c. on a very short interval containing $x=2$

$$\text{ave r.o.c. on } [a,b] = \frac{f(b) - f(a)}{b - a} = \frac{5^b - 5^a}{b - a} = \frac{5^b - 5^2}{b - 2}$$

$a=2$, h	$b=a+h$	ave. r.o.c
.01	2.01	40.5614
.0001	2.0001	40.2391
.000001	2.000001	40.2359
-.000001	1.999999	40.2359
-.0001	1.9999	40.2327
-.01	1.99	39.9138

gets closer to 0 ↓

gets closer to 0 ↑

estimate:

$$f'(2) \approx 40.2359$$

actual value:

$$f'(2) = \ln(5) \cdot 5^2 = 40.235947\dots$$