Study sheet for SHORTCUTS TO DIFFERENTIATION

Let's review the rules we have learned in sections 3.1 – 3.3 on particular examples.

constant multiple rule $\frac{d}{dx}\left(c\cdot f(x)\right) = c\cdot\frac{d}{dx}\left(f(x)\right)$

in other notation: $\left(c\cdot f(x)\right)' = c\cdot f'(x)$

sum rule $\frac{d}{dx}\left(f(x)+g(x)\right) = \frac{d}{dx}f(x) + \frac{d}{dx}g(x)$

$\left(f(x)+g(x)\right)' = f'(x)+g'(x)$

Example (a) $y = 5+\pi$ $\quad y' = \frac{dy}{dx} = 0$ (derivative of a constant function)

(b) $y = 5-4x$ $\quad y' = \frac{dy}{dx} = -4$ (derivative of linear function)

(c) $y = x^8$ $\quad y' = 8x^7,\quad y'' = 8\cdot 7\cdot x^6$

power rule (p any real number) $\frac{d}{dx}\left(x^p\right) = px^{p-1}$

(d) $z = \sqrt[3]{x} - 8^x$ $\quad z' = \frac{dz}{dx} = \frac{1}{3}x^{-2/3} - 8^x\cdot\ln(8)$

exponential rule ($a>0$ real fixed number) $\frac{d}{dx}\left(a^x\right) = a^x\cdot\ln(a)$

(e) $w = 5\ln(t)$ $\quad w' = \frac{dw}{dt} = 5\cdot\frac{1}{t}$

logarithm rule $\frac{d}{dx}\left(\ln(x)\right) = \frac{1}{x}$

Try (f) $y = 5x^3 - 3\sqrt[4]{x} = \ldots$ (rewrite as difference of power functions)

$y' =$

(g) $r(t) = 5 - 4\cdot 3^x$

$r'(t) =$

We can combine these with the chain rule to compute derivatives of composite functions:

chain rule Given $y = f(u)$, $u = g(x)$

$$\frac{dy}{dx} = \frac{df}{du}\cdot\frac{du}{dx}$$

$$y' = f'(u)\cdot u'$$

$\frac{df}{du}$ means the derivative of f with respect to the variable u. Any other variable is viewed as a constant.

Examples: (h) $y = (3x+1)^5$

$u = 3x+1 \rightarrow u' = 3$

$y = f(u) = u^5 \longrightarrow f'(u) = 5u^4$

$$\frac{dy}{dx} = f'(u)\cdot u' = 5u^4\cdot 3 = 5(3x+1)^4\cdot 3$$

(i) $y = \sqrt[3]{5-x^2}$

$u = 5-x^2 \longrightarrow u' = -2x$

$f(u) = \sqrt[3]{u} = u^{1/3} \longrightarrow f'(u) = \frac{1}{3}u^{-2/3}$

$$\frac{dy}{dx} = \frac{df}{du}\cdot\frac{du}{dx} = \frac{1}{3}\left(5-x^2\right)^{-2/3}\cdot(-2x)$$

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(j) $y = 7 \cdot 2^{t^3 - 5t}$

$u = t^3 - 5t \longrightarrow u' = 3t^2 - 5$

$f(u) = 7 \cdot 2^u \longrightarrow f'(u) = 7 \cdot 2^u \cdot \ln(2)$

$$\frac{dy}{dt} = \frac{df}{du} \cdot \frac{du}{dt} = 7 \cdot 2^{t^3-5t} \cdot \ln(2) \cdot (3t^2 - 5)$$

(k) $y = 3x - \ln(8x + 8^x) = 3x - f(u)$

$u = 8x + 8^x$

$f(u) = \ln(u)$

$$y' = 3 - \frac{1}{8x + 8^x} \cdot (8 + 8^x \cdot \ln(8))$$

(l) $z = \sqrt[3]{1 + x^2} - 5 \cdot \ln(2 - x^3) = f(u) - s(w)$

$u = 1 + x^2$

$f(u) = \sqrt[3]{u} = u^{1/3}$

$w = 2 - x^3$

$s(w) = 5 \ln(w)$

$$\frac{dz}{dx} = \frac{df}{du} \cdot \frac{du}{dx} - \frac{ds}{dw} \cdot \frac{dw}{dx}$$

$$= \frac{1}{3} (1 + x^2)^{-2/3} \cdot 2x - 5 \cdot \frac{1}{2 - x^3} \cdot (-3x^2)$$

Now you work on these problems:

(m) $Q = 200(5 + t^2)^8$ $\qquad \frac{dQ}{dt} =$

$u = \ldots$ $\qquad f(u) = \ldots$

$u' =$ $\qquad f'(u) =$

(n) $P = \ln(5t + e^t)$ $\qquad \frac{dP}{dt} =$

(o) $R(x) = 5x - 3 \cdot 2^{x^4 - x}$ $\qquad \frac{dR}{dx} =$

(p) $f(x) = \ln(1 - 3 \cdot e^{4x})$ $\qquad \frac{df}{dx} =$

(q) $g(t) = t^2 - 3 + (2 - 5t)^3 + \ln(3t + 1) - e^{7t}$

$\frac{dg}{dt} =$

Also: Solve problems #33, #41 in Section 3.3.

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