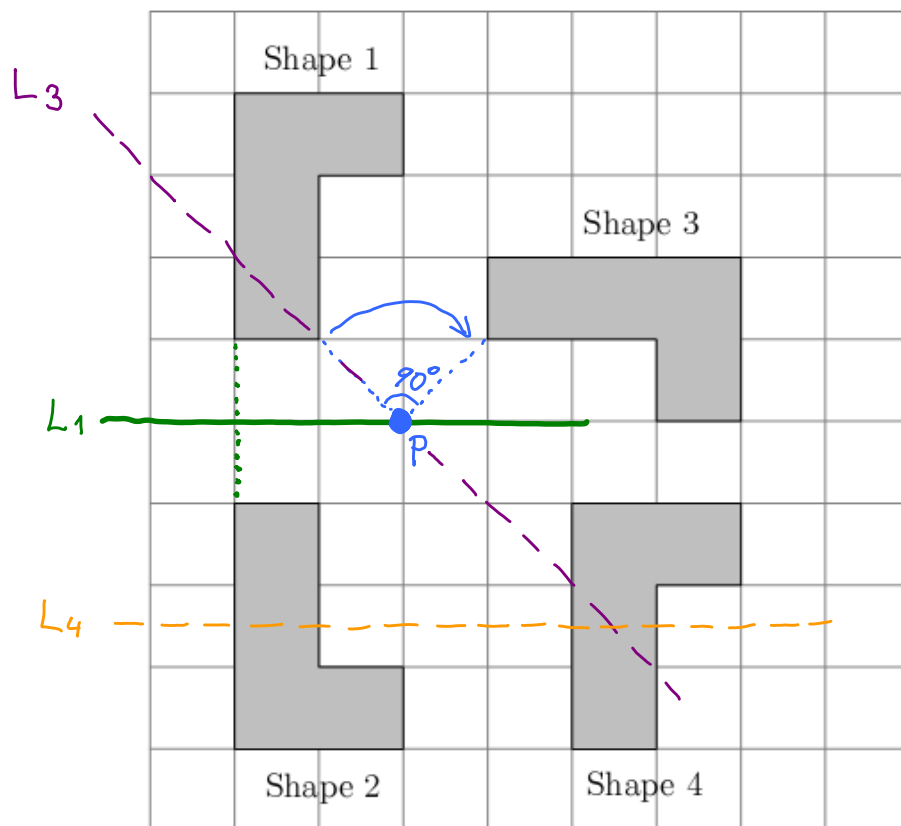


Directions: Answer all questions on the test paper. Show your work. There are many questions, spend your time accordingly.

1. (16 points) Using the graph below, answer the following. For rotations mark the center of rotation and tell the angle and direction, for (glide) reflections draw the axis of reflection.



(a) Determine a single transformation that will take Shape 1 to Shape 2.

Reflection along line L_1

(b) Determine a single transformation that will take Shape 1 to Shape 3.

90° clockwise rotation around the point P

(c) Determine a single transformation that will take Shape 2 to Shape 3.

Reflection along line L_3

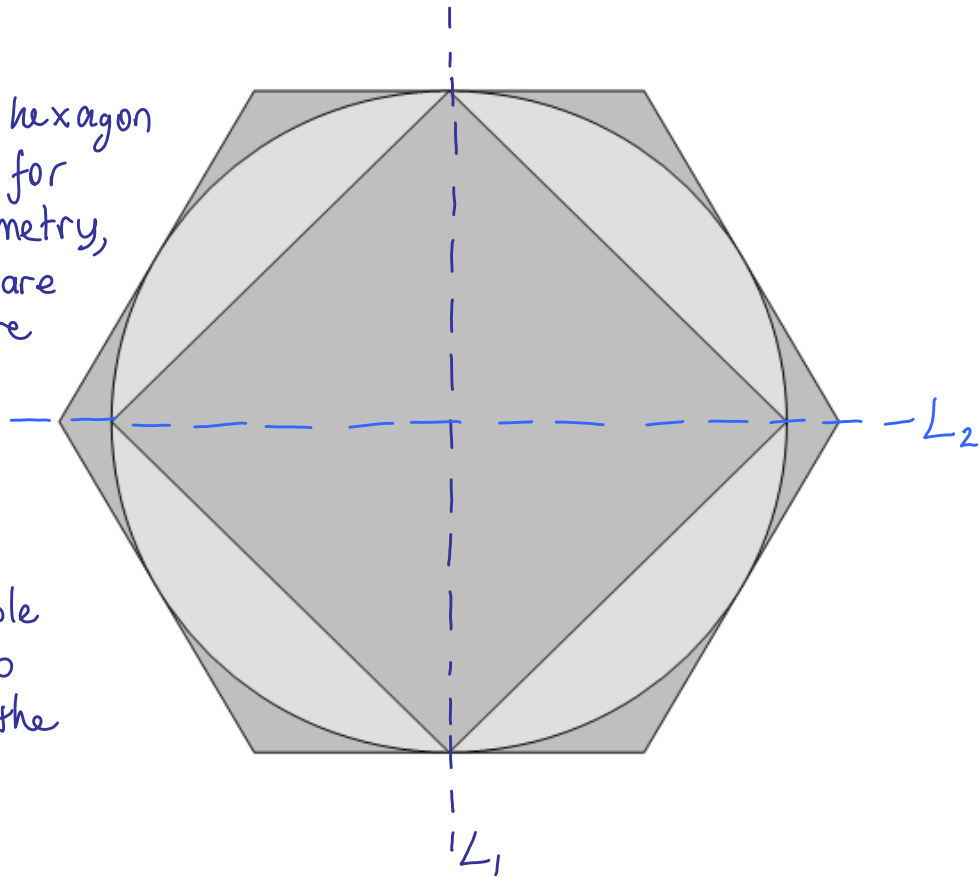
(d) Determine a single transformation that will take Shape 2 to Shape 4.

Glide-reflection along line L_4

2. (10 points) Determine all the symmetries of the following design.

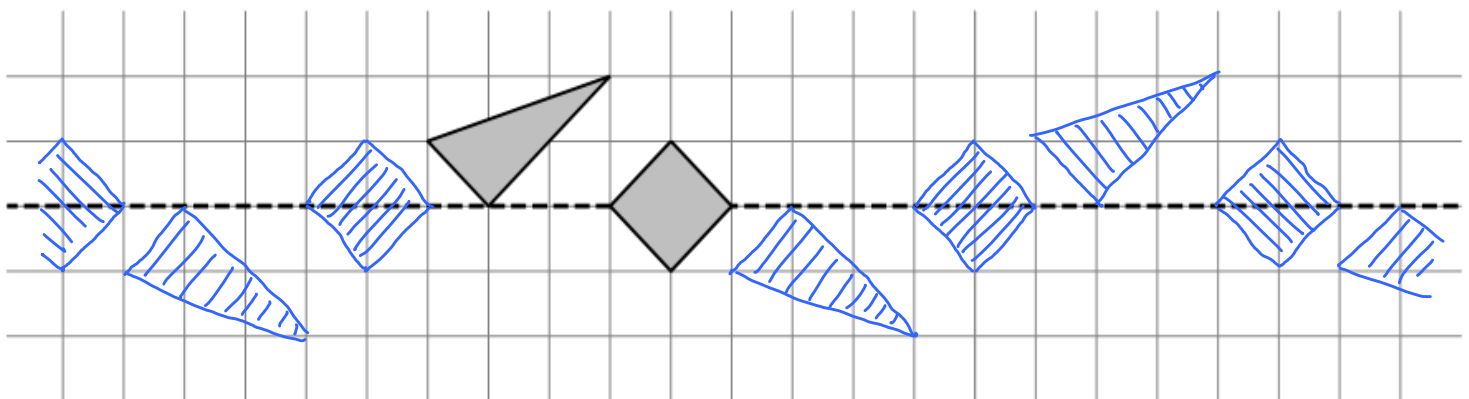
(a) Draw axes for reflection symmetry.

Even though a hexagon has more axes for reflection symmetry, due to the square inside, there are only two lines for reflection symmetry since the whole figure has to overlap with the reflection.



(b) The design has 2-fold rotation symmetry because the first time the design overlaps with itself is after rotating it 180° around its center.

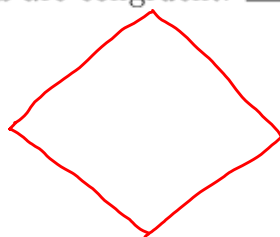
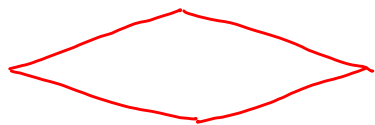
3. (8 points) Draw a design with glide-reflection symmetry using the following pattern and having the given reflection axis and the translation being 5 units.



4. (15 points) Indicate TRUE or FALSE for each statement. If the statement is false, give an explanation or a counterexample.

(a) Any two equilateral triangles are similar: TRUE

(b) Any two rhombi having same side lengths are congruent: FALSE



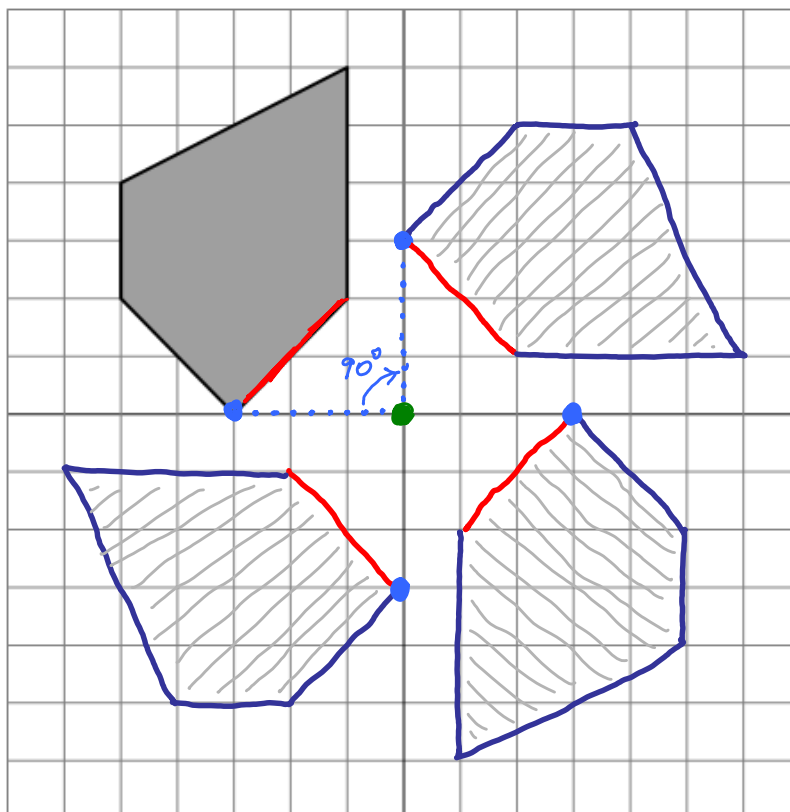
Both have same side lengths, but they are not congruent.

(c) A triangle with side lengths 4, 5, 6 is similar to a triangle with side lengths 7, 8, 9: FALSE

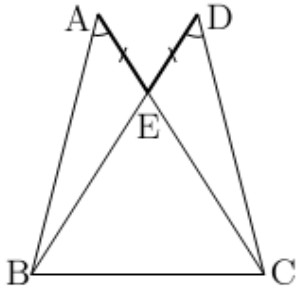
If they were similar, there should be one scaling factor, but $\frac{7}{4} \neq \frac{8}{5}$ ($\neq \frac{9}{6}$) hence they can't be similar.

5. (8 points) Start with the following asymmetric shape, and complete it to make a design which has a 4-fold rotation symmetry. Use the origin as the center of rotation symmetry.

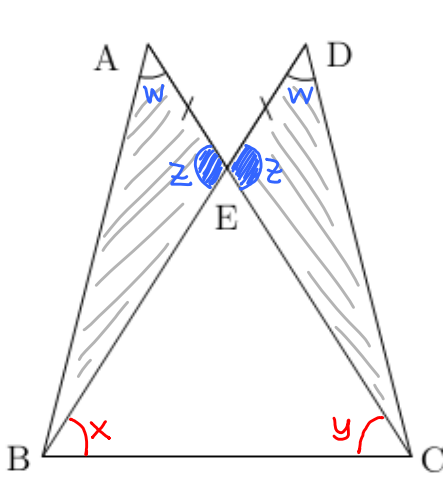
$$\frac{360^\circ}{4} = 90^\circ$$



6. (12 points) Show why the following is true using a two-column elementary proof format. (Don't mark the first figure, use the second one)



Given: $\overset{w}{\angle BAC} = \overset{w}{\angle BDC}$ and $AE = DE$,
 Show that: $\overset{x}{\angle DBC} = \overset{y}{\angle ACB}$.

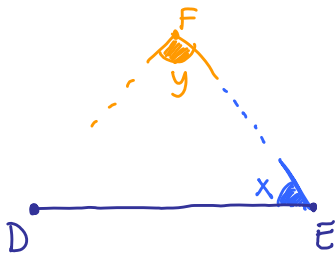


- $\triangle AEB \cong \triangle DEC$ by ASA congruence:
 - A: $\angle BAE = \angle CDE = w$ given
 - S: $AE = DE$ given
 - A: $\angle AEB = \angle DEC = z$ vertical angles
- Hence we get: (1) $AB = DC$
 (2) $BE = CE$
 (3) $\angle ABE = \angle DCE$
- $\angle DBC = \angle ACB$ $\triangle BEC$ is isosceles by (2)
these are the base angles

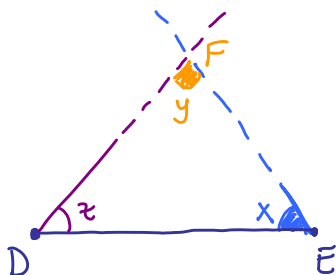
7. (10 points) Explain why SAA is a congruence relation. In other words, given consecutive side, angle, angle information, why does it determine a unique triangle up to congruence? Use a figure to illustrate your explanation.

Given SAA information, we want to show that

there is a unique way to find the location for the third vertex F.



Since we know two angles, we can compute the third: $z = 180 - (x + y)$ (we converted SAA into ASA)



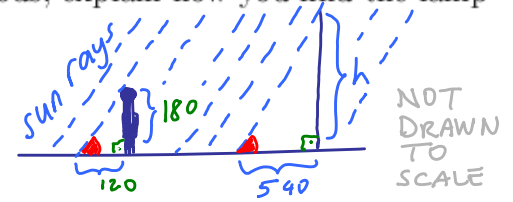
Extend two rays from D and E that make angles x and z. They intersect at a single point, that has to be the third vertex of the triangle.

8. (12 points) Tom is 180cm tall. At 3pm in the afternoon Tom's shadow measures 120cm on the ground. He measures a lamp post's shadow as 540cm. Using two different methods, explain how you find the lamp post's height. State the name of the method you use.

(a) Using scale factor method:

$$120 \times \text{scaling factor} = 540 \rightarrow \text{scaling factor} = \frac{540}{120}$$

$$180 \times \text{scaling factor} = h \rightarrow h = 180 \times \frac{540}{120}$$



3rd way:
Setting up a proportion

$$\frac{540}{120} = \frac{h}{180}$$

$$h = 180 \times \frac{540}{120}$$

(b) Using relative sizes method:

relative size: Tom's height is $\frac{180}{120}$ times his shadow's length.

The post and its shadow should have the same relative size:

$$\frac{h}{540} = \frac{180}{120} \rightarrow h = \frac{180}{120} \times 540$$

9. (9 points) John wants to estimate the height of his family's Christmas tree. He uses two different methods:

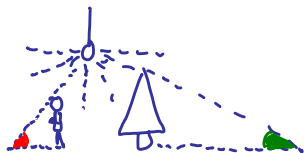
Method 1. He turns on the ceiling light, measures the length of the tree's shadow and the length of his own shadow. Using similarity he estimates the height of the tree.

Method 2. There is a table next to the tree which is 1 yard tall, stretches out his arm, with one eye closed he holds a ruler vertically to measure how tall the tree appears on the ruler, and then uses similarity to estimate the height of the tree.

Discuss which method gives a better estimate. Why is the other method faulty in this case?

Method 2 gives a better estimate.

In method 1, the location of the tree and of John will affect the length of the shadow, so most probably we won't be able to find similar triangles.



Sun rays come parallel
but not the light rays from the lamp.