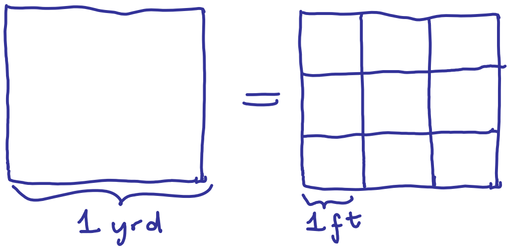
MTH 202 – Spring 2008 Section 4 DURUSOY

Exam 3 Form D – April 2008 – 100 points SOLUTIONS

1. (10 points) John wants to buy a tablecloth to cover a round table of area 26 square feet. At the store he sees a round tablecloth of area 3 square yards. Find the area of the tablecloth in square feet. Will it cover his table?



area: 1 square-yard = 9 square-feet

Hence

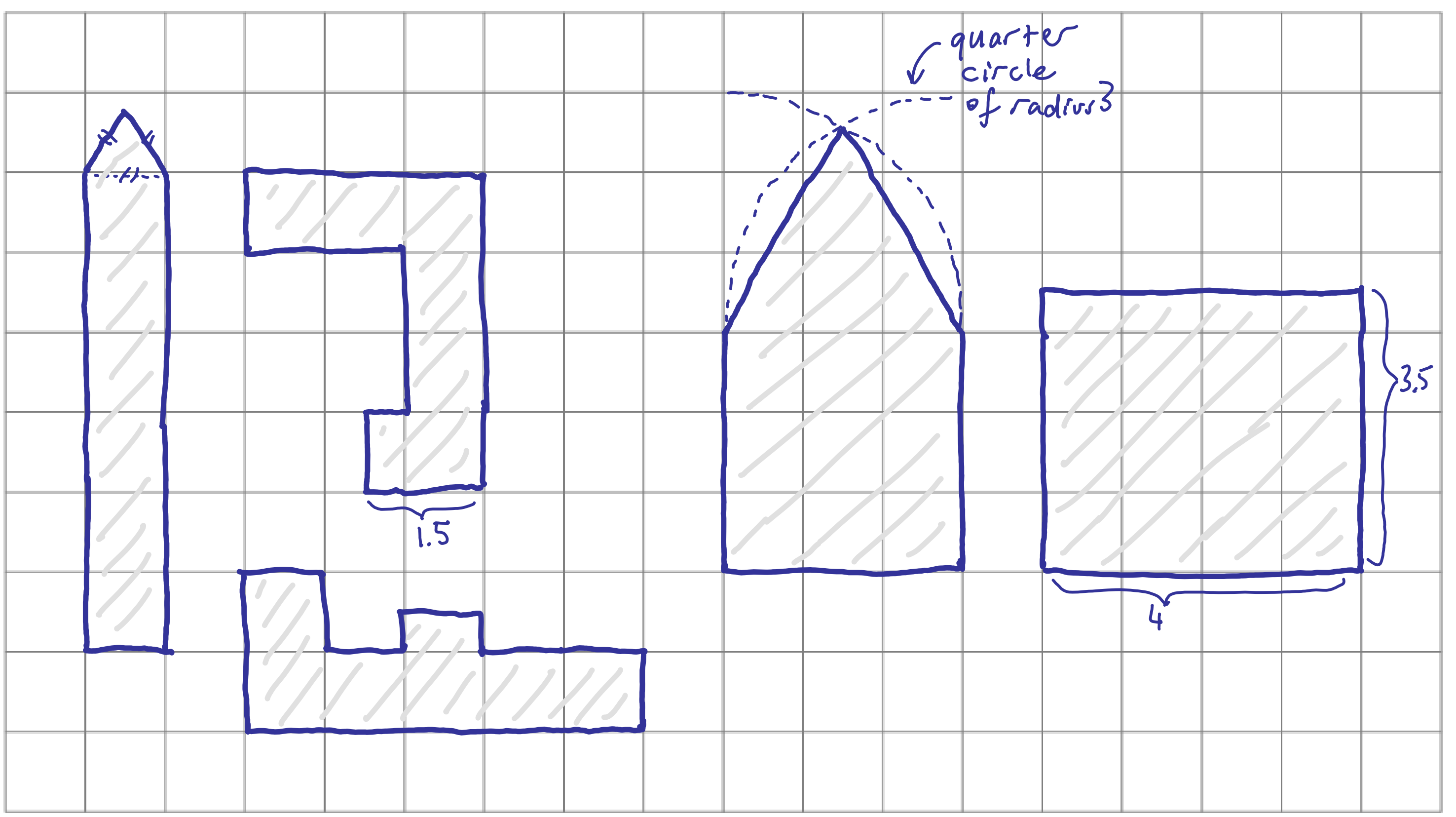
3 square yards = $3 \times 9 = 27$ square feet

$27 > 26$, so it will cover his table.

Note: we didn't need the radius, but to find the radius solve $27 = \pi r^2$

$r = \sqrt{\frac{27}{\pi}}$ feet

2. (8 points) Draw two different regions that have a perimeter of 15 centimeters.



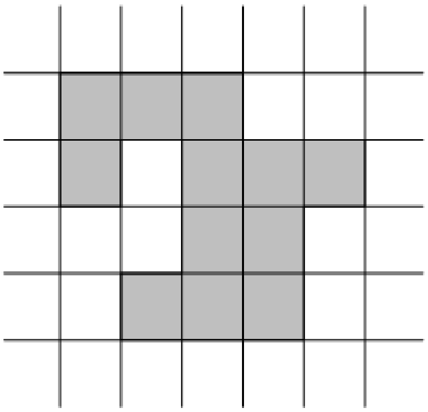
3. (6 points) For an olympic indoors swimming pool, give examples of 1-, 2-, and 3-dimensional aspects. Include appropriate units.

1D : Length of pool in feet

2D : Surface area of pool in square feet

3D : amount of water in pool in gallons

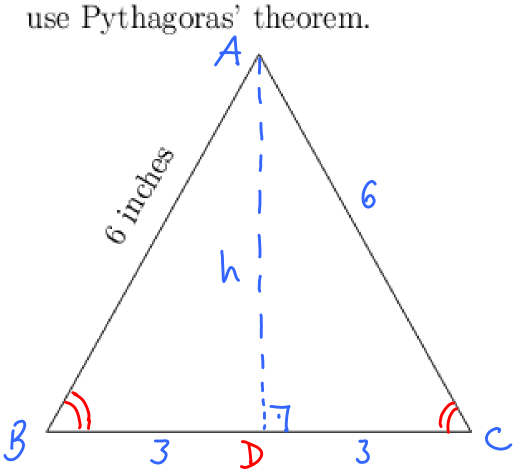
4. (8 points) Give two examples of errors a students could make while trying to calculate the perimeter of an object on a grid. (Give one short sentence for each)



(1) A student could count every side of each box in the shaded area,
but they should count only the exterior sides.

(2) A student could count the number of boxes along the boundary, but this is also wrong.

5. (12 points) An equilateral triangle has side length 6 inches. Find its area. Explain briefly why you can use Pythagoras' theorem.



$h^2 + 3^2 = 6^2$

$h^2 = 36 - 9 = 27$

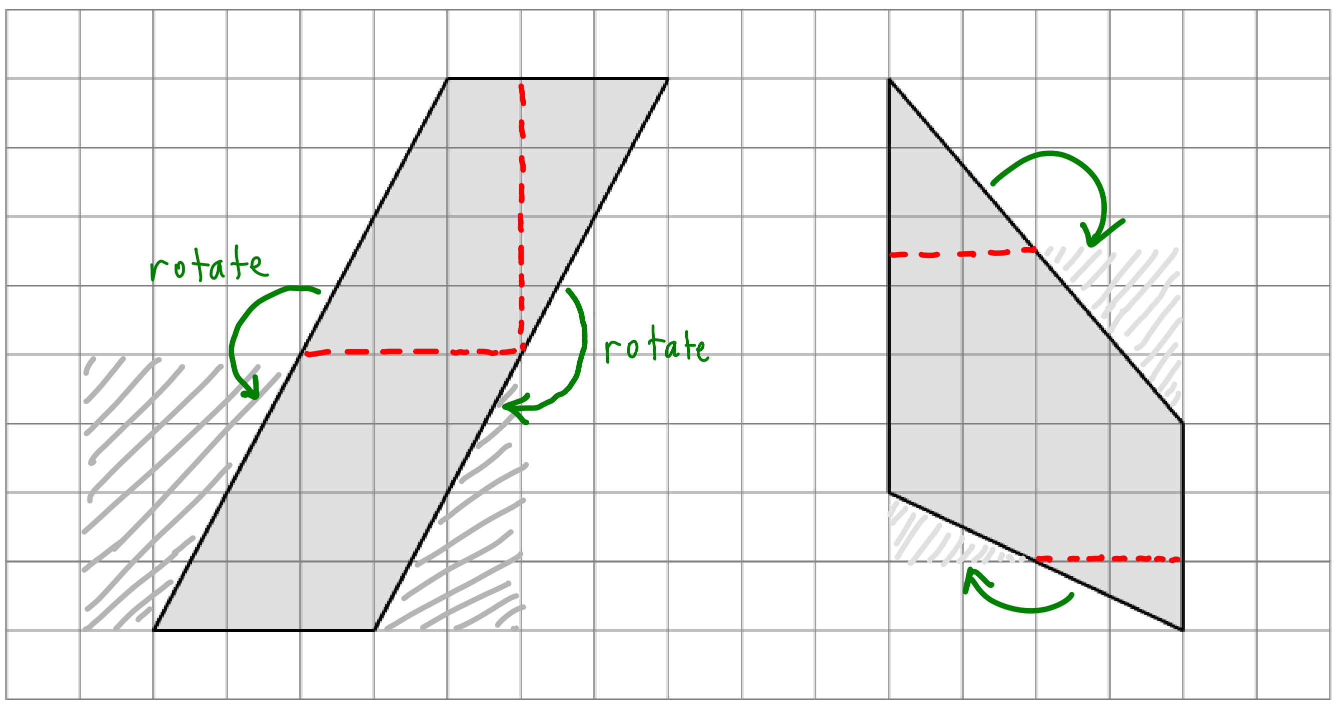
$h = \sqrt{27}$

Area $= \frac{1}{2}(\text{base} \times \text{height}) = \frac{1}{2}(6 \times \sqrt{27}) = 3\sqrt{27}$

$= 9\sqrt{3}$

Explanation: Let D = midpoint of BC
since AB = AC, AD is the angle bisector and
also perpendicular to BC.
hence ADC is a right triangle.

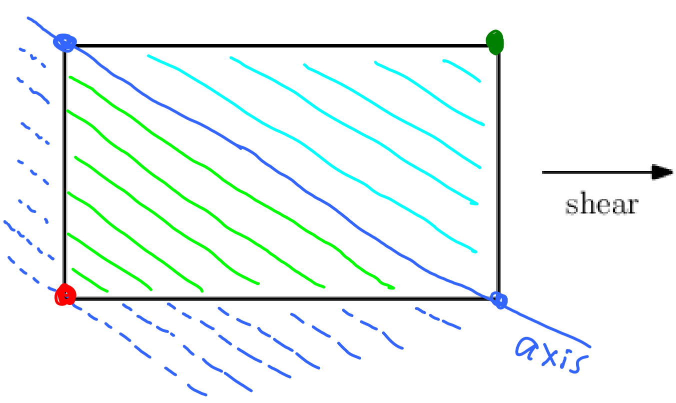
6. (10 points) Cut and rearrange the following shapes to obtain rectangles of the same area. Show where you have cut by a dotted line, and shade the new location, with an arrow explaining the displacement.

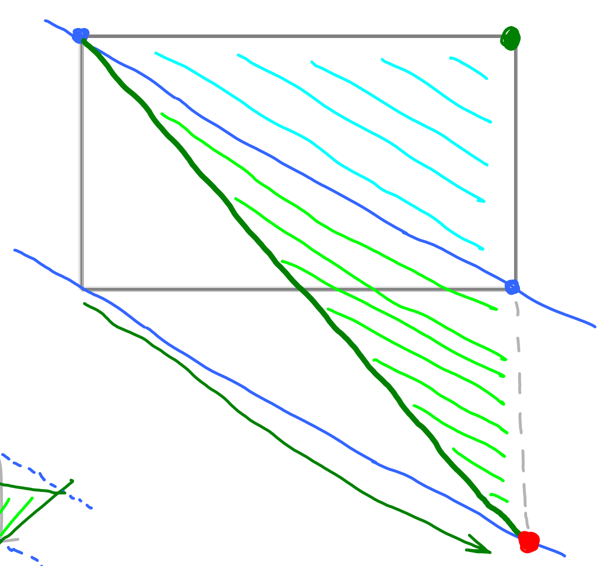


There are other ways to cut, too.

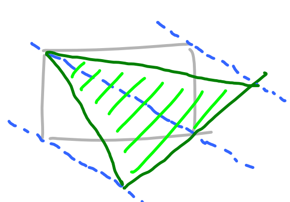
7. (10 points) Shear the following rectangle to get a triangle. First choose an axis to shear parallel to. Draw the axis and at least 3 slices on the left picture, draw the result after shearing on the right. Hint: can you get a triangle if the axis is parallel to an edge?

Axis has to be parallel to a diagonal of the rectangle.

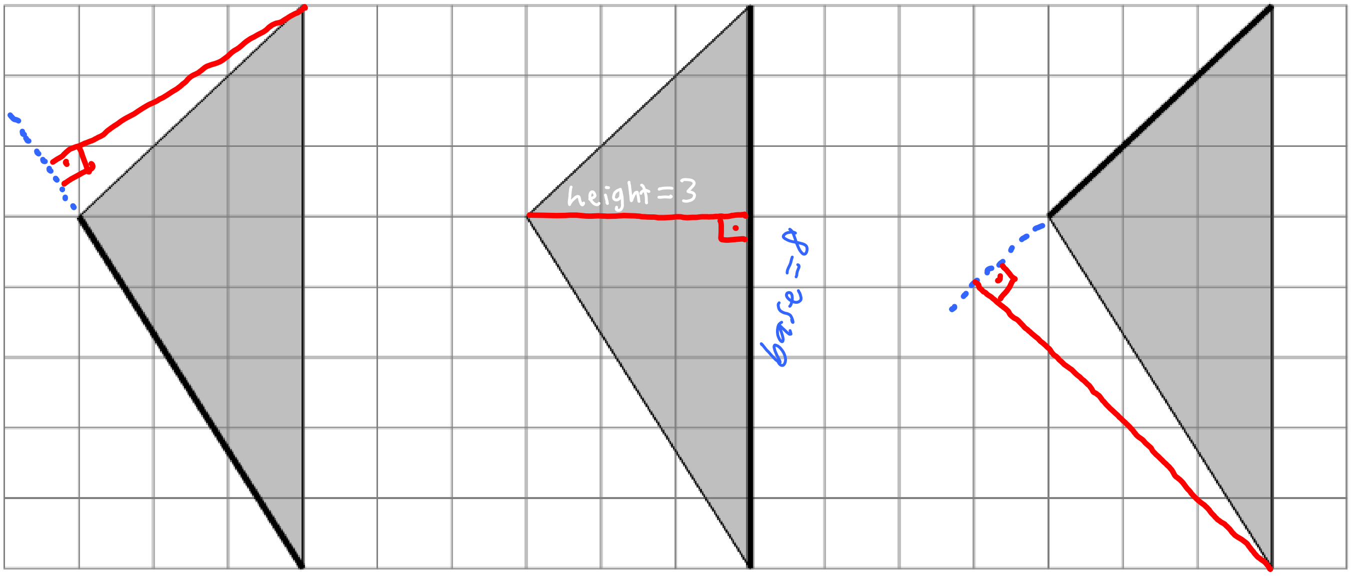




another way:

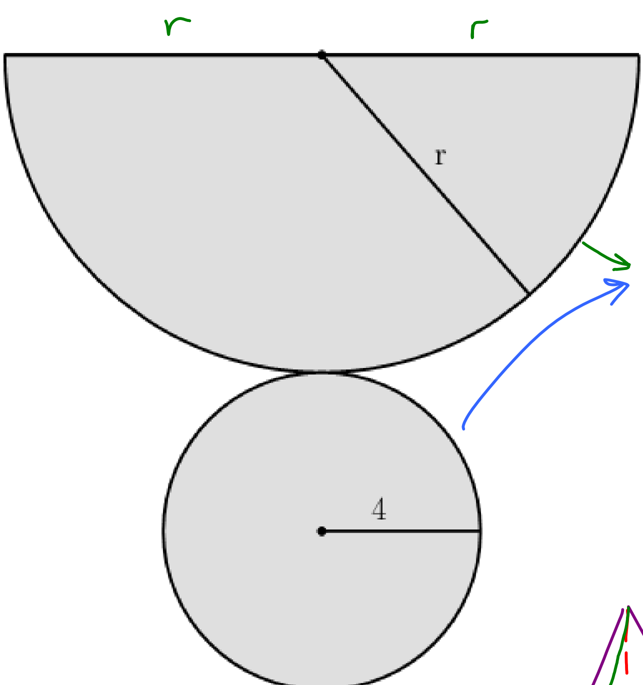


8. (12 points) The following is three copies of the same triangle. Draw heights corresponding to the thicker sides. Compute the are of this triangle.



Area = $\frac{1}{2}(8 \times 3) = \frac{1}{2}(24) = 12$

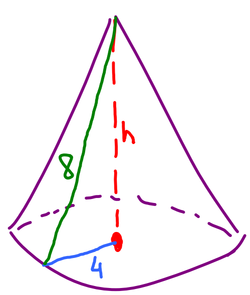
9. (12 points) A teacher wants to make a cone for demonstration using a half circle for the top and a circle with radius 4 for the bottom. What will be the height for the cone?



should have same arc length
not counting the diameter

$$2 \cdot \pi \cdot 4 = \frac{1}{2}(2\pi r)$$

$$8\pi = \pi r \longrightarrow r = 8$$



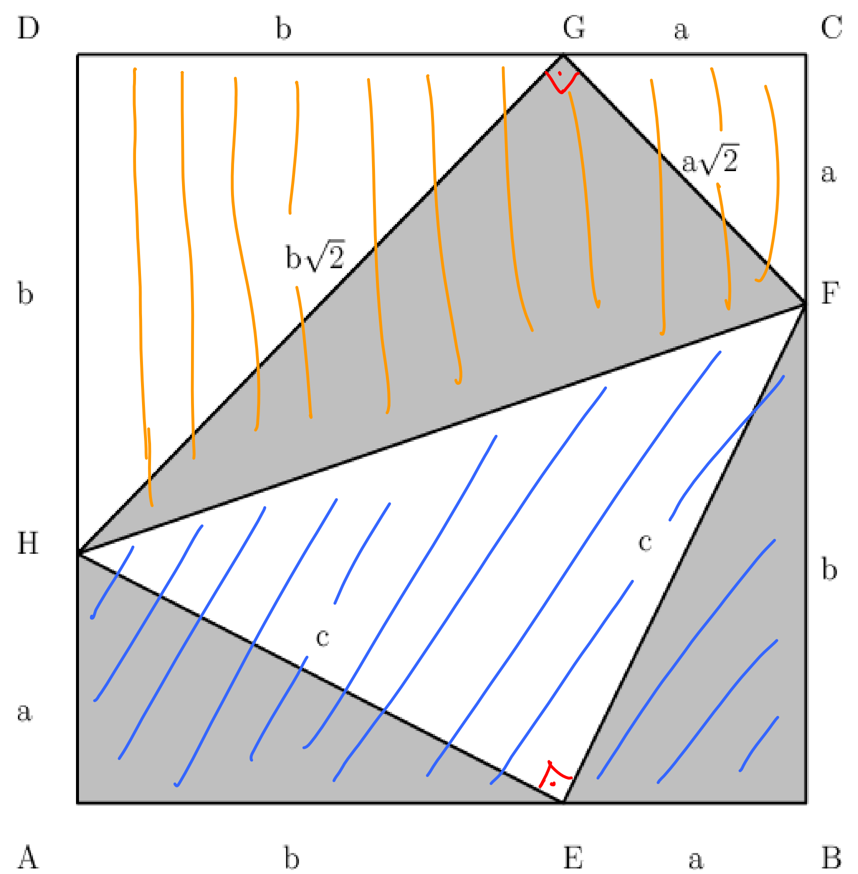
$$h^2 + 4^2 = 8^2$$

$$h^2 = 64 - 16 = 48$$

$$h = \sqrt{48} = 4\sqrt{3}$$

10. (12 points) Using the following picture give a proof of the Pythagorean theorem. Hint: first find which angles are congruent, which angles are 90 degrees.

ABCD is a square.



(a) Circle correct one: ABFH is congruent to CDHF after translation / reflection / rotation / glide-reflection

180° around center of ABCD

(b) Area of ABFH in terms of a, b, c:

sum of areas of 3 triangles:

$\frac{1}{2}ba + \frac{1}{2}c^2 + \frac{1}{2}ab$

$= ab + \frac{c^2}{2}$

(c) Area of CDHF in terms of a, b:

sum of areas of 3 triangles:

$\frac{1}{2}b^2 + \frac{1}{2}(a\sqrt{2} \cdot b\sqrt{2}) + \frac{1}{2}a^2$

$= \frac{1}{2}b + ab + \frac{1}{2}a^2$

(d) Set the two areas equal, simplify to get Pythagorean theorem.

$ab + \frac{c^2}{2} = ab + \frac{a^2}{2} + \frac{b^2}{2}$

subtract ab from both sides

$\frac{1}{2}(c^2) = \frac{1}{2}(a^2 + b^2)$

factor out $\frac{1}{2}$.

$c^2 = a^2 + b^2$

multiply both sides by 2