

**Math 202 Section 4**

**QUIZ 5 SOLUTIONS**

Feb 15, 2008

Show your work in all problems.

1. (a) Define *rotation*. Describe how points in the plane move for a given rotation (for a given center and angle of rotation). Does every point move?

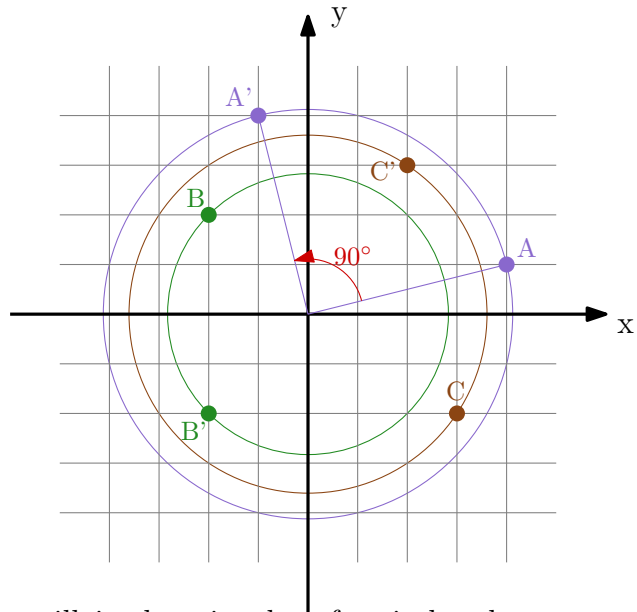
A rotation is a transformation of the plane, that fixes a point C and rotates all other points in the plane around the point C at a given angle. The points move along circles centered at C and the center is the only point that doesn't move.

(b) On the grid at the right, mark the points  $A(4, 1), B(-2, 2), C(3, -2)$ . Also plot the new points  $A', B', C'$  obtained after rotating the plane  $90^\circ$  counterclockwise.

$$A(4, 1) \mapsto A'(-1, 4)$$

$$B(-2, 2) \mapsto B'(-2, -2)$$

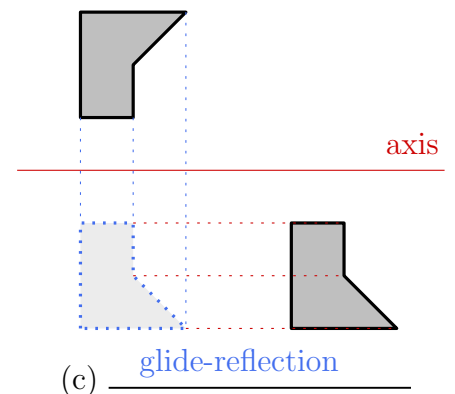
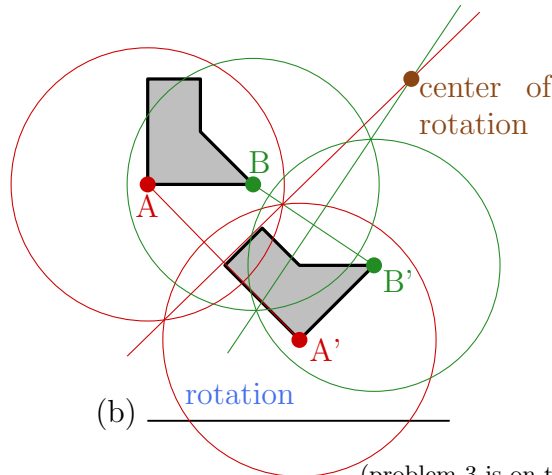
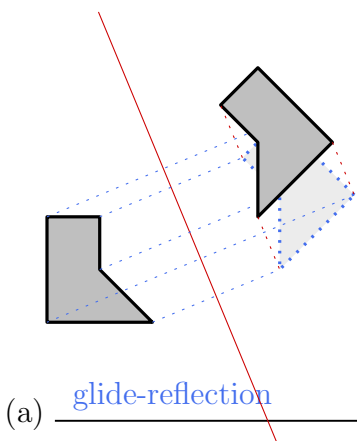
$$C(3, -2) \mapsto C'(2, 3)$$



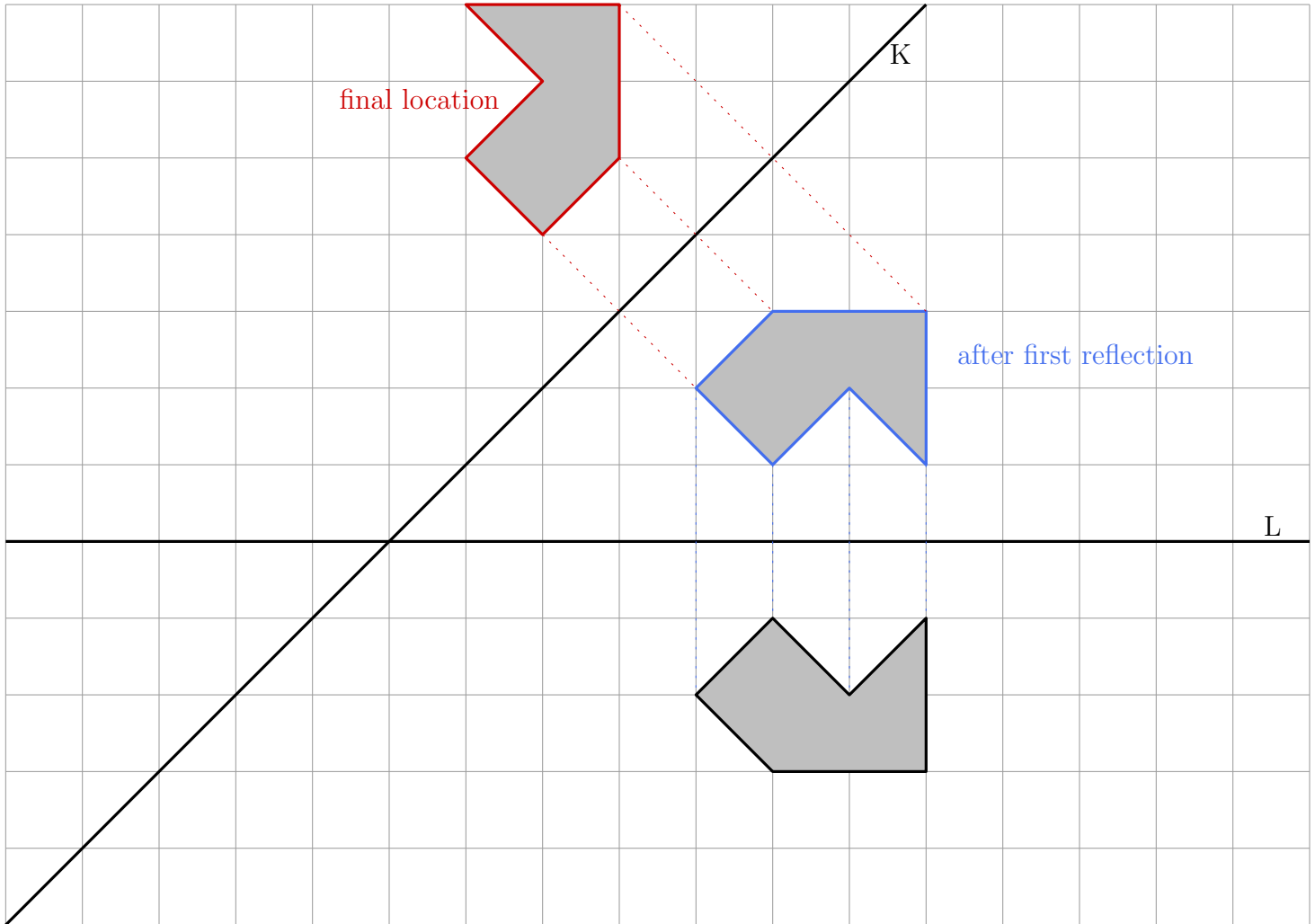
(c) If  $(p, q)$  is a point on the coordinate plane, what will its location be after it has been rotated  $90^\circ$  counterclockwise about the origin? Give the new coordinates, explain your answer briefly.

$(p, q)$  will move to  $(-q, p)$  after  $90^\circ$  counterclockwise rotation. Explanation: points on x-axis go to points on y-axis after rotation, so the x-coordinate which was the projection of a point to x-axis becomes its projection to y-axis. We also pay attention to signs, and get the result.

2. A *glide-reflection* is the end result of combining a reflection and then a translation in the direction of the line of reflection. Identify if the following are obtained by a single translation, rotation, reflection or glide-reflection. For a rotation estimate its center, and for a (glide-) reflection estimate its axis.



3. (a) For the given shape below, apply the following transformations consecutively: reflection along the line L, then reflection along the line K.

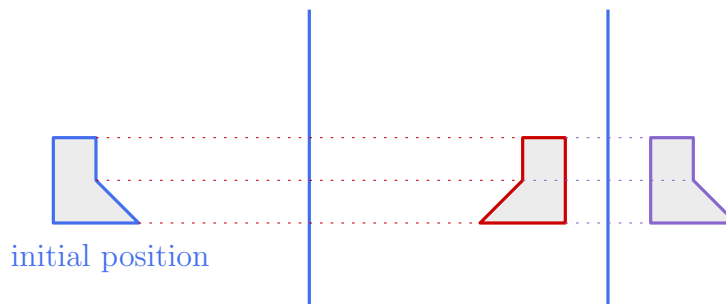


(b) Comparing the initial and final locations, could you obtain the same result using one single transformation? Give details.

These two consecutive reflections correspond to a counterclockwise  $90^\circ$  rotation around the intersection of the two axes.

(c) Is your conclusion from part (b) always correct? Or can there be other outcomes after two different reflections are applied consecutively? Hint: consider when the two axes are parallel.

When the two axes intersect, it will always give a rotation, but if we reflect consecutively along parallel axes, the final result is a translation of the original, not a rotation. Example:



Note: use a polygon with no symmetries. "The Boot" has no symmetries.