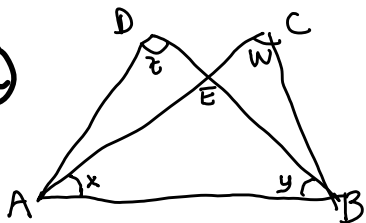
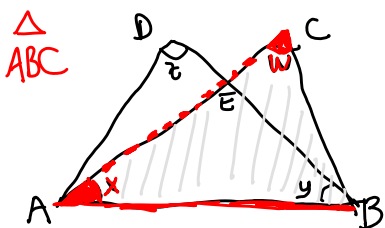


① Fill the gaps in the following proofs:

②



Given $x=y$ and $z=w$
Show that $AD = BC$



$\triangle ABC \cong \triangle BAD$ by **AAS** congruence:

A • $w = z$

given

A • $x = y$

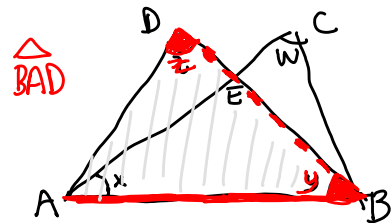
given

S • $AB = BA$

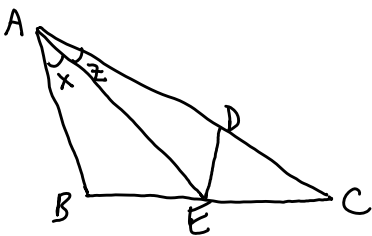
common side

hence $AD = BC$

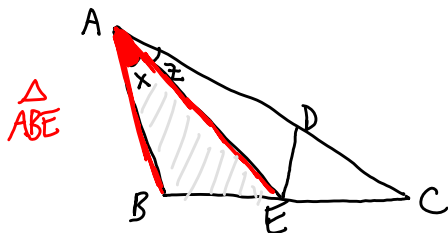
corresponding side lengths



③



Given $AB = AD$ and $x = z$
Show that $BE = DE$



$\triangle ABE \cong \triangle ADE$ by **SAS** congruence:

S • $AB = AD$

given

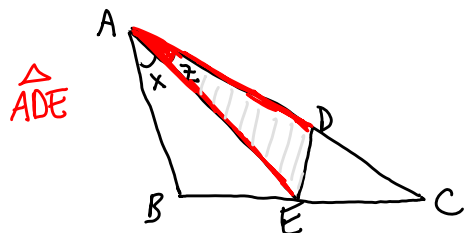
A • $x = z$

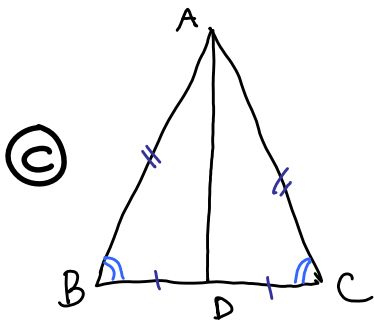
given

S • $AE = AE$

common edge

hence $BE = DE$





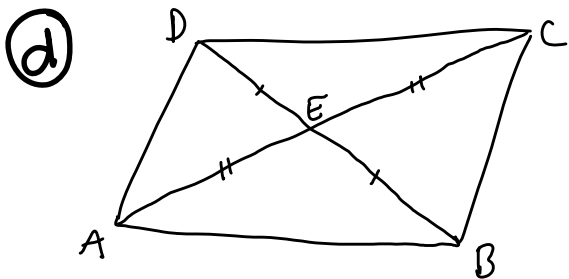
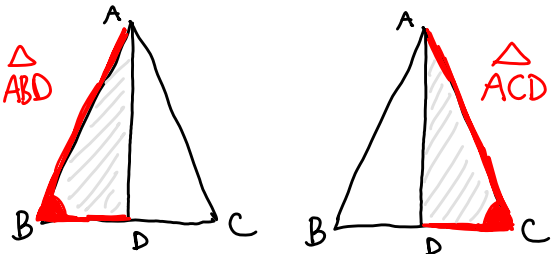
Given $AB = AC$ and $BD = DC$
 show that AD bisects the angle $\angle BAC$

$\angle ABC = \angle ACB$ $\triangle ABC$ is isosceles

$\triangle ABD \cong \triangle ACD$ by **SAS** congruence

- $AB = AC$ given
- $\angle ABC = \angle ACB$
- $BD = DC$

hence $\angle BAD = \angle CAD$



Given $AE = EC$, $DE = EB$
 Show that $ABCD$ is a parallelogram.

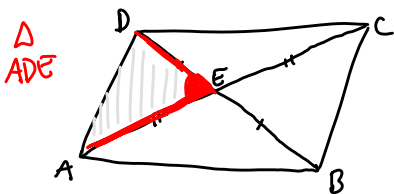
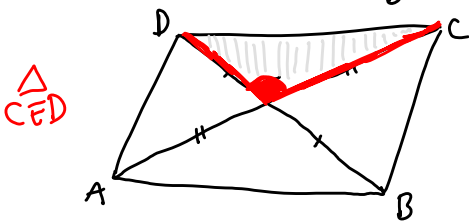
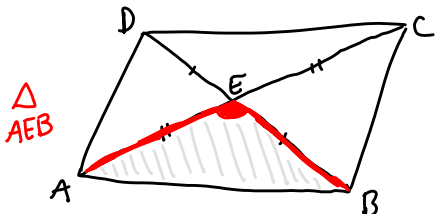
$\triangle AEB \cong \triangle CED$ by **SAS** congruence:

- $AE = CE$ given
- $\angle AEB = \angle CED$ vertical angle theorem
- $BE = DE$ given

hence $\angle ABE = \angle CDE$

$AB \parallel CD$

parallel postulate and vertical angle theorem



Similarly $\triangle ADE \cong \triangle CBE$

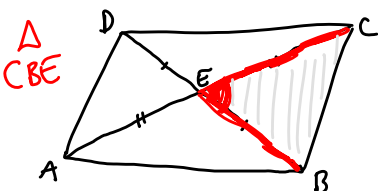
- $AE = CE$
- $\angle AED = \angle CEB$
- $ED = EB$

by **SAS** congruence:

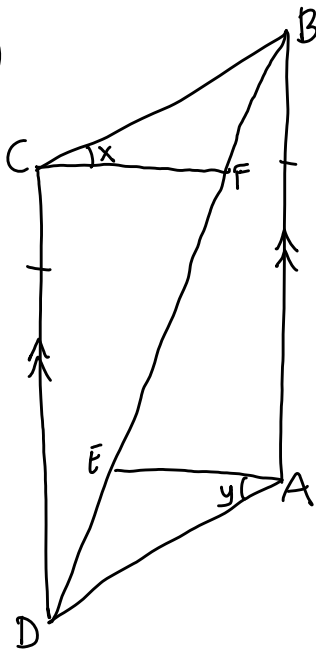
- given
- vertical angle theorem
- given

hence $\angle DAE = \angle BCE$
 $AD \parallel BC$

parallel postulate and vertical angle theorem



2

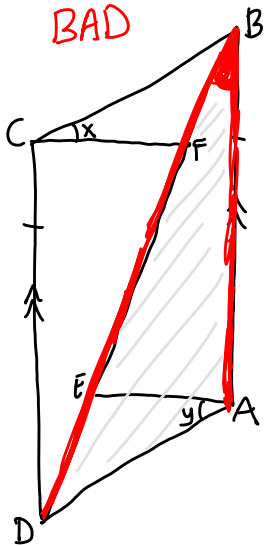


Given $AB \parallel DC$, $AB = DC$, $x = y$
 Show that $AE = CF$.
 Hint: first show $AD = CB$.

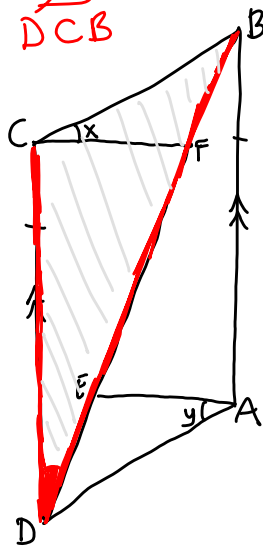
to show this, we choose two triangles including one of these as an edge, that we can show to be congruent.

first, observe $\angle ABD = \angle CDB$ using $AB \parallel DC$ and DB is a transversal.

$\triangle BAD$



$\triangle DCB$



$\triangle BAD \cong \triangle DCB$ by SAS:

$AB = CD$

$\angle ABD = \angle CDB$

$BD = DB$

hence $AD = CB$

$\angle ADB = \angle CBD$

given
parallel postulate
common side

$\triangle AED \cong \triangle CFB$ by ASA:

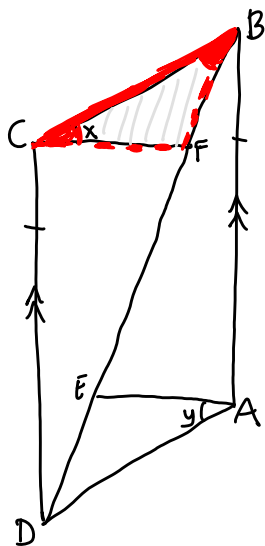
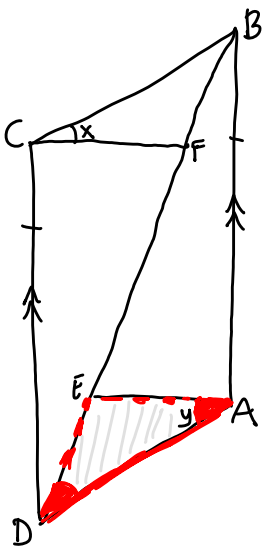
$x = y$

$AD = CB$

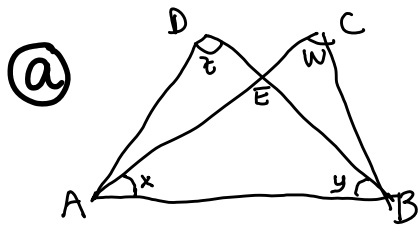
$\angle ADB = \angle CBD$

hence $AE = CF$

given
} shown above



① Fill the gaps in the following proofs:

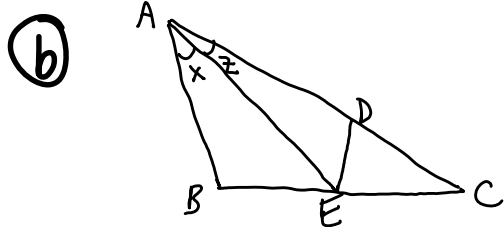


Given $x = y$ and $z = w$
 Show that $AD = BC$

$\triangle ABC \cong \triangle BAD$ by ... congruence:

-
-
- $AB = BA$ common side

hence $AD = BC$ corresponding side lengths

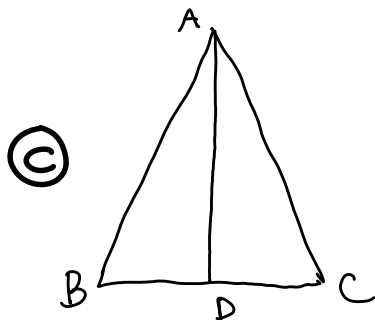


Given $AB = AD$ and $x = z$
 Show that $BE = DE$

$\triangle ABE \cong \triangle ADE$ by ... congruence:

- $AB = AD$ given
-
-

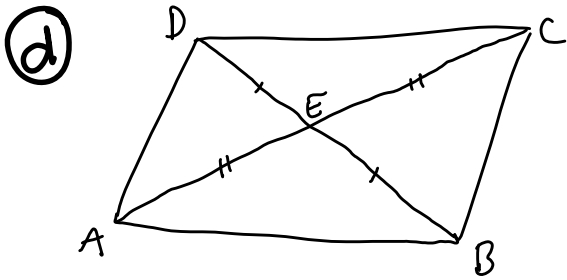
hence $BE = DE$



Given $AB = AC$ and $BD = DC$
 show that AD bisects the angle $\angle BAC$

$\angle ABC = \angle ACB$ $\triangle ABC$ is isosceles
 $\triangle ABD \cong \triangle ACD$ by ... congruence

- $AB = AC$ given
-
-



Given $AE = EC$, $DE = EB$
 Show that $ABCD$ is a parallelogram.

$\triangle AEB \cong \triangle CED$ by ... congruence:

- $AE = CE$ given
- $\angle AEB = \angle CED$...
-

hence $\angle ABE = \angle CDE$

$AB \parallel CD$ parallel postulate and vertical angle theorem

by ... congruence:
 given

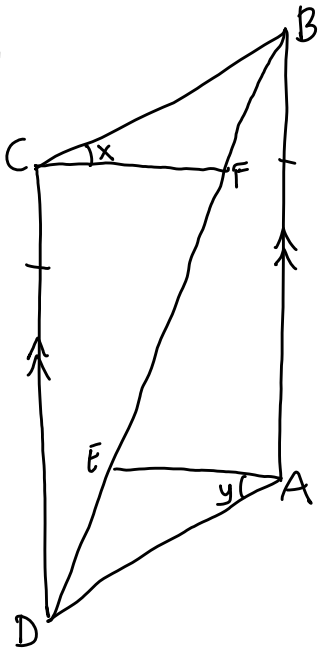
Similarly $\triangle ADE \cong \triangle BCE$

- $AE = CE$
-
-

hence $\angle DAE = \angle BCE$
 $AD \parallel BC$

parallel postulate and vertical angle theorem

2



Given $AB \parallel DC$, $AB = DC$, $x = y$

Show that $AE = CF$.

Hint: first show $AD = CB$.

$\triangle BAD \cong \triangle DCB$ by ...