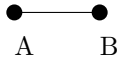
Geometric Constructions using Straightedge and Compass

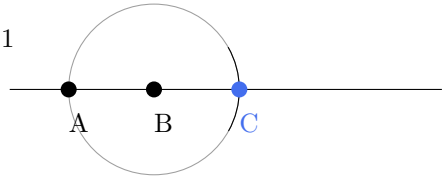
A geometric construction is an accurate drawing of a shape using only the following tools:
1. Straightedge: A ruler without markings on it. Can be used to draw straight lines. Can't be used for measuring.
2. Compass: Can be used to draw circles, or arcs. The radius can be fixed by placing the pin at one special point and the marker at another special point.

Example 1. Given a unit distance line segment, construct a line segment of length 3 units.



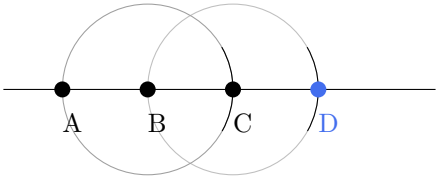
Given two points.

Step 1



Draw a longer line though A and B. Measure the distance between A and B with the compass and draw a circle centered at B with that radius. The new intersection gives us a point C which is two units from A.

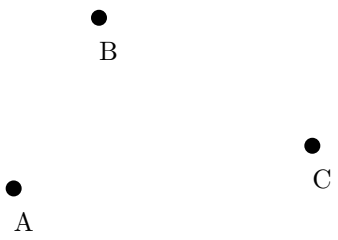
Step 2



Using the same measurement, draw a circle centered at C with that radius. The new intersection gives us a point D which is three units from A.

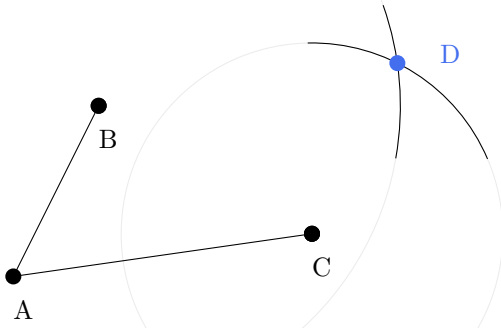
Remark. In the above figure then line through the two intersection points of the circles will intersect the line segment BC in the middle perpendicularly. The fact that the two circles have the same radius is crucial.

Example 2. Given three points (not collinear), form the parallelogram with those as vertices.
All we need to do is to find the exact location of the fourth corner. Then with a straightedge we can draw the parallelogram. As in other examples, we will use intersections of lines and circles to construct auxiliary points to help us find the missing corner.



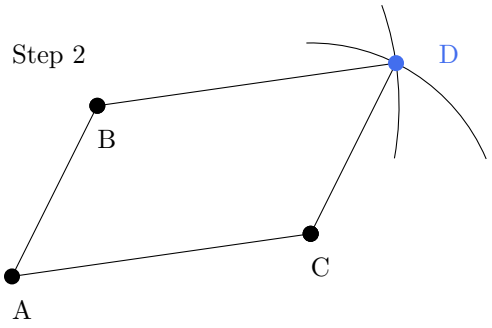
Given three points.

Step 1



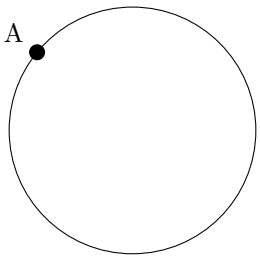
Measure the length of AB with the compass, draw a circle of that radius centered at C. Next, measure the length of AC and draw a circle centered at B. They will meet at the desired point. Reason: the distance from D to B is by construction same as the length AC, and similarly the distance from D to C is by construction same as the length AB.

Step 2

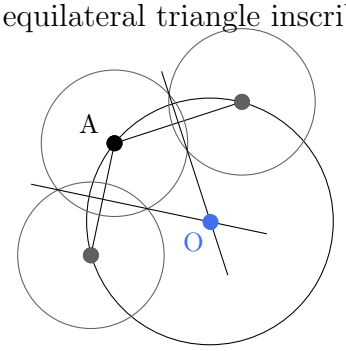


Remark. In the above figure if the lengths of AB and AC were the same, we would have constructed a rhombus, and the line AD would be the angle bisector for the angle BAC.

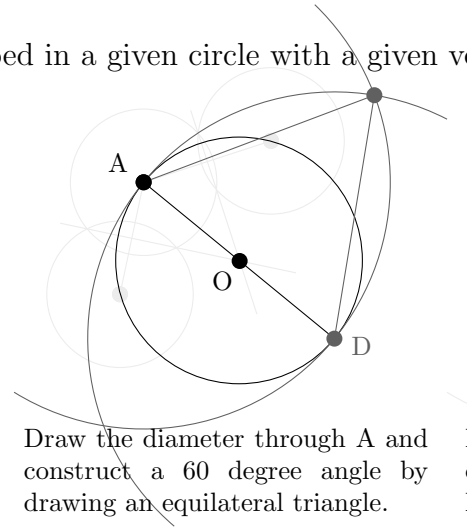
Example 3. Finding an equilateral triangle inscribed in a given circle with a given vertex.



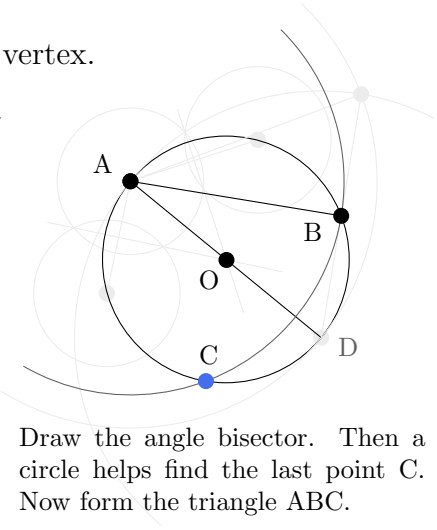
Given circle and point.



Draw two chords for the circle. Their perpendicular bisectors will meet at the circle's center.



Draw the diameter through A and construct a 60 degree angle by drawing an equilateral triangle.



Draw the angle bisector. Then a circle helps find the last point C. Now form the triangle ABC.

Remark. Drawing the angle bisector at the last step is the same as drawing a line to the intersection of the original circle with the auxiliary equilateral triangle because the angle ADB is 60 and the angle ABD is 90, hence the angle DAB is 30. ABC is an isosceles triangle by construction, but CBA is 60 degrees, so it is equilateral.