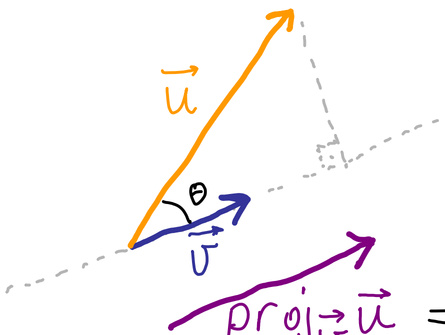
Math 234 QUIZ 1 July 9, 2007

1. For the vectors $\mathbf{u} = \langle 2, 5, -1\rangle$ and $\mathbf{v} = \langle 2, 1, 4\rangle$ find the vector projection of $\mathbf{u}$ onto the direction of $\mathbf{v}$.



$$|\text{proj}_v u| = |u| \cdot \cos\theta = \frac{|u|\cdot|v|\cdot\cos\theta}{|v|} = \frac{u \bullet v}{|v|}$$

$$\text{unit vector parallel to } v = \frac{v}{|v|}$$

$$\text{proj}_{\vec{v}}\vec{u} = \frac{u\bullet v}{|v|}\cdot\frac{v}{|v|} = \frac{(2\cdot 2 + 5\cdot 1 + (-1)\cdot 4)\cdot\langle 2,1,4\rangle}{2^2+1^2+4^2}$$

$$= \frac{5}{21}\cdot\langle 2,1,4\rangle = \left\langle \frac{10}{21}, \frac{5}{21}, \frac{20}{21}\right\rangle$$

2. If for non-zero vectors $\mathbf{u}\cdot\mathbf{v} = \mathbf{u}\cdot\mathbf{w}$, does $\mathbf{v}$ and $\mathbf{w}$ have to be the same vector? Give a proof or disprove giving a counterexamlple.

No, v and w don't have to be the same vector.
Idea: dot product only uses the lengths and angle between two vectors. As long as $|v|\cdot\cos\theta_1 = |w|\cdot\cos\theta_2$, we'll have $u\bullet v = u \bullet w$.

Counterexample: two distinct vectors having same dot product with u:

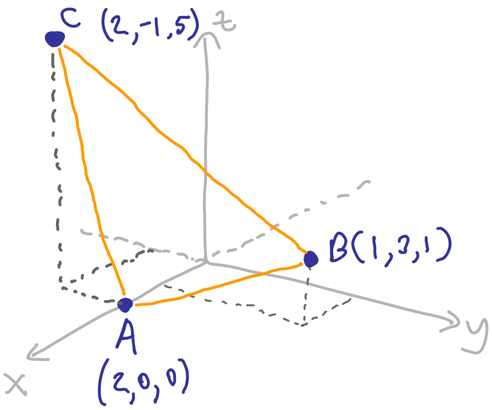
$u = \langle 1,2,1\rangle$

$$\left.\begin{aligned} v &= \langle 2,0,0\rangle \\ w &= \langle 0,1,0\rangle \end{aligned}\right\}$$

$u\cdot v = u\cdot w = 2$
but clearly $v \neq w$
also $|v| \neq |w|$

Hence with dot product we don't have "cancellation property".

3. Find the area of the triangle with vertices at $(2,0,0), (1,3,1), (2,-1,5)$.



$\vec{u} = \overrightarrow{AB} = \langle 1-2, 3-0, 1-0 \rangle = \langle -1, 3, 1 \rangle$

$\vec{v} = \overrightarrow{AC} = \langle 2-2, -1-0, 5-0 \rangle = \langle 0, -1, 5 \rangle$

recall: $|\vec{u} \times \vec{v}|$ = area of parallelogram spanned by $\vec{u}, \vec{v}$

$= 2 \cdot \text{area}(\widehat{ABC})$

$$u \times v = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & 3 & 1 \\ 0 & -1 & 5 \end{vmatrix} = (3 \cdot 5 - 1 \cdot (-1))\vec{i} - (-1 \cdot 5 - 1 \cdot 0)\vec{j} + (-1 \cdot (-1) - 3 \cdot 0)\vec{k}$$

$$= \langle 16, 5, 1 \rangle$$

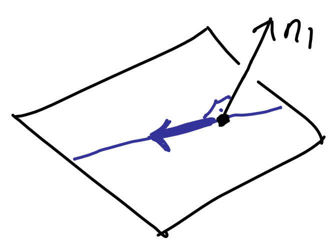
$$\text{area}(\widehat{ABC}) = \frac{1}{2} |u \times v| = \frac{1}{2} \cdot \sqrt{16^2 + 5^2 + 1^2} = \frac{\sqrt{282}}{2}$$

4. Find the equation of the plane through $(1,2,3)$ and perpendicular to the line of intersection of the planes $5x - 3y = 4$ and $x + y - 3z = 2$.

Recall: $ax + by + cz + d = 0$ defines a plane whose normal is $\langle a, b, c \rangle$.

$n_1 = \langle 5, -3, 0 \rangle$

$n_2 = \langle 1, 1, -3 \rangle$



the direction vector of any line in a plane is perpendicular to the normal.

the direction of the line of intersection is perpendicular to the normals of both planes, hence parallel to $\vec{n_1} \times \vec{n_2}$:

$$n_1 \times n_2 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 5 & -3 & 0 \\ 1 & 1 & -3 \end{vmatrix} = \langle 9, 15, 8 \rangle$$

Plane through $(1,2,3)$ with normal $\langle 9, 15, 8 \rangle$:

$$9(x-1) + 15(y-2) + 8(z-3) = 0$$