Math 234.201 US 07 QUIZ 2 July 18, 2007

1. Given $w = xy + 2cos(y)$ and $\boxed{x = \frac{u^2}{v}}$, $\boxed{y = v^3 + \ln(u)}$, find $\frac{\partial w}{\partial u}$ in two ways:

(a) using chain rule:

$$\frac{\partial w}{\partial u} = \frac{\partial w}{\partial x}\cdot\frac{dx}{du} + \frac{\partial w}{\partial y}\cdot\frac{dy}{du}$$

$$= y \cdot \frac{2u}{v} + (x - 2\sin y)\cdot\frac{1}{u}$$

(b) first substituting x and y into w

$$w = \frac{u^2}{v}\cdot(v^3 + \ln(u)) + 2\cos(v^3 + \ln(u))$$

$$\frac{\partial w}{\partial u} = \frac{2u}{v}\cdot(v^3 + \ln(u)) + \frac{u^2}{v}\cdot\frac{1}{u} - 2\sin(v^3 + \ln(u))\cdot\frac{1}{u}$$

(c) evaluate $\left.\frac{\partial w}{\partial u}\right|_{(2,\pi)}$ "$u=v$"

$$\left.\begin{matrix} u = 2 \\ v = \pi \end{matrix}\right\} \quad \begin{matrix} x = \frac{4}{\pi} \\ y = \pi^3 + \ln(2) \end{matrix}$$

$$\left.\frac{\partial w}{\partial u}\right|_{(2,\pi)} = (\pi^3 + \ln(2))\cdot\frac{4}{\pi} + \left(\frac{4}{\pi} - 2\sin(\pi^3 + \ln(2))\right)\cdot\frac{1}{2}$$

2. Find the directional derivative of $f(x, y) = 3xy + 4sin(y)$ at $P_0(3, 0)$ in the direction of $\mathbf{A} = \langle 5, 12\rangle$. Hint: first find unit vector in the direction of $\mathbf{A}$.

$$u = \frac{A}{|A|} = \left\langle \frac{5}{13}, \frac{12}{13}\right\rangle$$

$$\nabla f = \langle f_x, f_y\rangle = \langle 3y, 3x + 4\cos(y)\rangle$$

$$\nabla f|_{(3,0)} = \langle 0, 9+4\rangle = \langle 0, 13\rangle$$

$$(D_u f)_{(3,0)} = \nabla f|_{(3,0)}\cdot u = \langle 0, 13\rangle\cdot\left\langle\frac{5}{13}, \frac{12}{13}\right\rangle = 12$$

3. For the surface $z^2 = x^2 + 2xy - y^2$ find the equation of the tangent plane at $P_0\,(1, 1, -\sqrt{2})$. Hint: First express the surface as $f(x, y, z) = 0$.

$$f(x,y,z) = x^2 + 2xy - y^2 - z^2$$

$$\nabla f = \langle f_x, f_y, f_z \rangle = \langle 2x + 2y,\ 2x - 2y,\ -2z \rangle$$

$$\vec{n} = \nabla f \big|_{(1,1,-\sqrt{2})} = \langle 4,\ 0,\ 2\sqrt{2} \rangle$$

tangent plane: $4(x-1) + 0(y-1) + 2\sqrt{2}\left(z - (-\sqrt{2})\right) = 0$

4. Find the equation of the tangent line to the curve of intersection of the surfaces $z^2 = x^2 + 4y^2$ and $x^2 - y^2 - z^2 + 9 = 0$ tangent at $P_0(1, 1, 3)$.

$$f(x,y,z) = x^2 + 4y^2 - z^2 \qquad \nabla f = \langle 2x, 8y\ -2z \rangle$$

$$g(x,y,z) = x^2 - y^2 - z^2 + 9 \qquad \nabla g = \langle 2x, -2y, -2z \rangle$$

$$\vec{n}_1 = \nabla f \big|_{(1,1,3)} = \langle 2, 8, -6 \rangle$$

$$\vec{n}_2 = \nabla g \big|_{(1,1,3)} = \langle 2, -2, -6 \rangle$$

direction of tangent line $= \vec{v} = \vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} i & j & k \\ 2 & 8 & -6 \\ 2 & -2 & -6 \end{vmatrix}$

$$= \langle -60,\ 0,\ -20 \rangle$$

line equation: $\vec{r}(t) = (1 - 60t,\ 1,\ 3 - 20t)$

this can also be written as: $\vec{r}(t) = (1 + 3t, 1, 3 + t)$

since $(3,0,1) \parallel (-60,0,-20)$