Last example revisited:

Ex: does the limit $\lim_{(x,y)\to(0,0)} \frac{x^3 \cdot y}{x^2+y^2}$ exist?

hint: $\frac{4xy^2}{x^2+y^2}$ has limit 0 at the origin (example 3, p978)

We failed to show in class that the limit doesn't exist, reason:
Actually the limit exists:

Since we're interested in limit at the origin, assume $\delta < 1$

$0 \le y^2 \longrightarrow x^2 \le x^2+y^2 \xrightarrow[|x|<1]{\text{assume}} |x^3| \le x^2+y^2$

$$\left|\frac{x^3}{x^2+y^2}\right| \le 1$$

hence $\left|\frac{x^3 y}{x^2+y^2}\right| \le |y|$ ☆ (now we could use multi variable sandwich theorem)

Given $\varepsilon > 0$, we want to find corresponding $\delta > 0$ so that

if $0 < \underbrace{|(x,y)-(0,0)|}_{\sqrt{x^2+y^2}} < \delta$, then $\left|\frac{x^3 y}{x^2+y^2} - 0\right| < \varepsilon$

guess for limit

☆ → $\left|\frac{x^3 y}{x^2+y^2} - 0\right| \le |y| = \sqrt{y^2} \le \underbrace{\sqrt{x^2+y^2}}_{\le \delta}$

hence if we choose $\delta = \min\{1, \varepsilon\}$, by definition

$$\underbrace{\lim_{(x,y)\to(0,0)} \frac{x^3 y}{x^2+y^2} = 0}$$

in SciLab, this is how you define and visualize a surface:

```
deff('[z]=Surf(x,y)', 'z=(x^3)*y/(x^2+y^2)');     // defines z as a function of x and y
t=-2.001 : 0.15 : 2;                              // tells the range of t and increment
fplot3d1(t, t, Surf);
```

observe that the surface gets flatter around origin