Modeling the role of lateral membrane diffusion in AMPA receptor trafficking along a spiny dendrite

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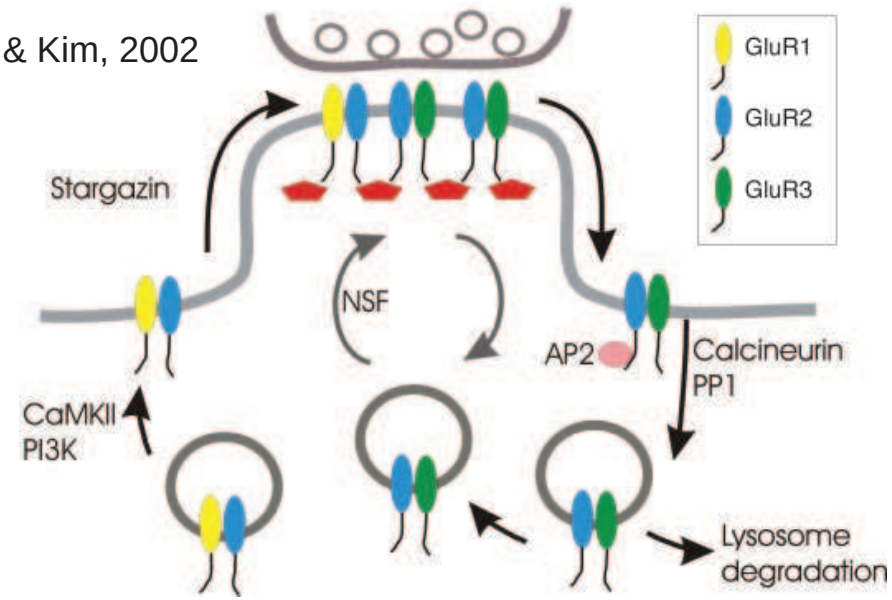
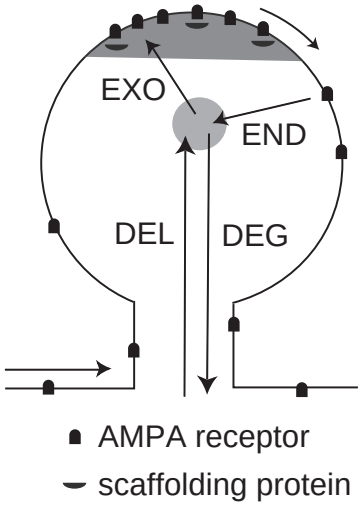
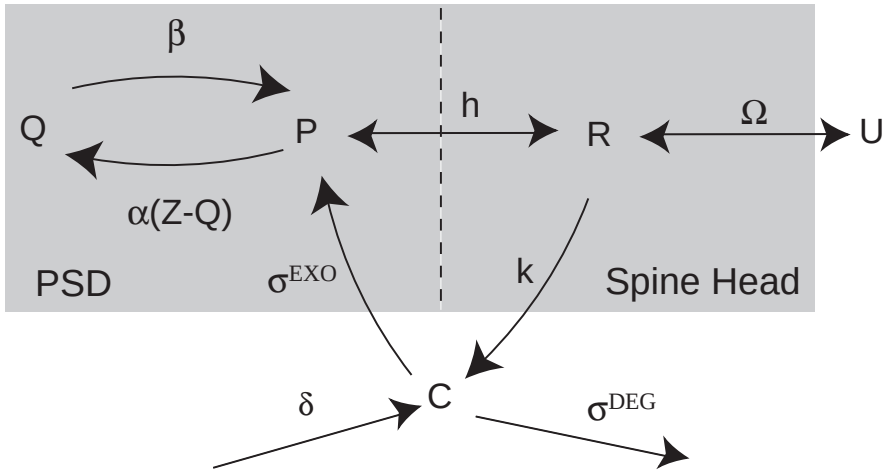
AMPA receptor trafficking at spines



- Surface AMPARs constitutively recycle with intracellular stores
- Laterally diffuse within postsynaptic membrane
- Crosslink to scaffolding proteins in PSD

Model of trafficking at a single spine

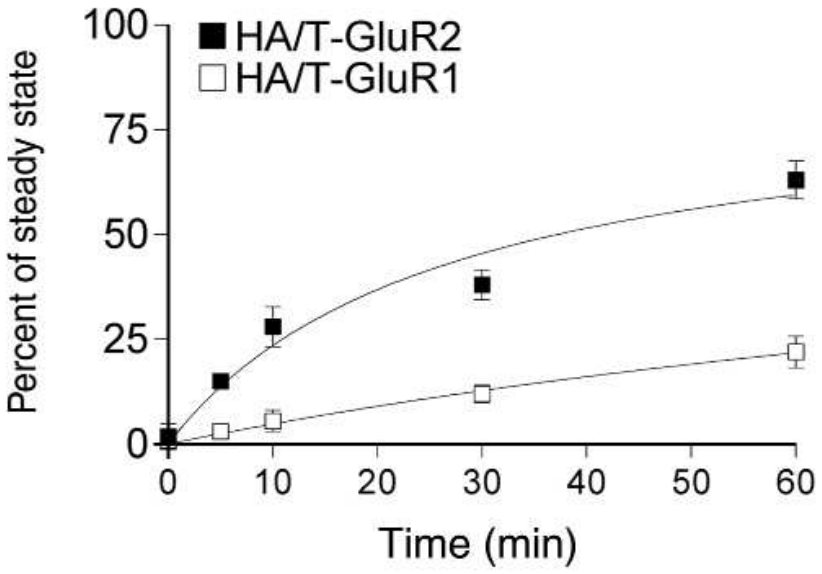




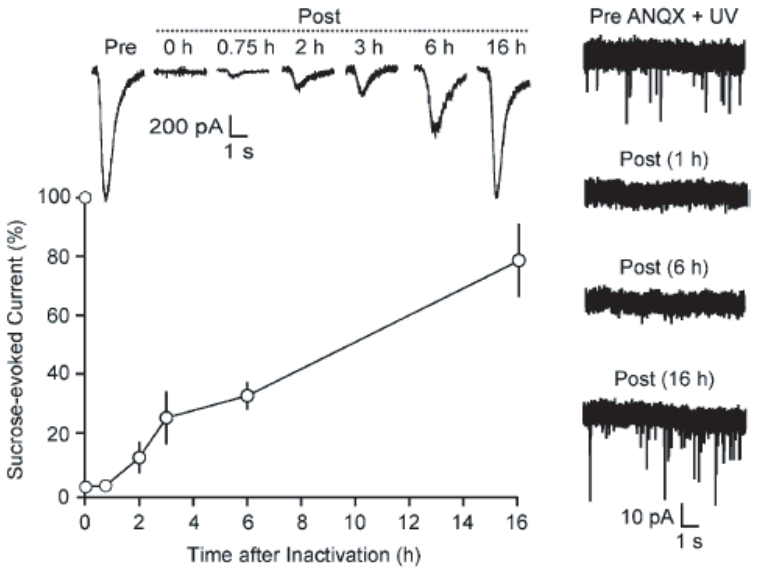
P, Q:	**unbound, bound receptor concentrations in PSD**
R, U:	**free receptor concentrations in spine head, dendrite**
C:	**number of intracellular receptors**
k, σ^{EXO}:	**rates of endocytosis, exocytosis**
σ^{DEG}, δ:	**rates of degradation, intracellular delivery**
h, Ω:	**hopping rates across boundary of PSD, spine neck**
α(Z-Q):	**rate of binding to scaffolding (Z = scaffolding concentration)**
β:	**rate of unbinding from scaffolding**

Fast or slow recycling?

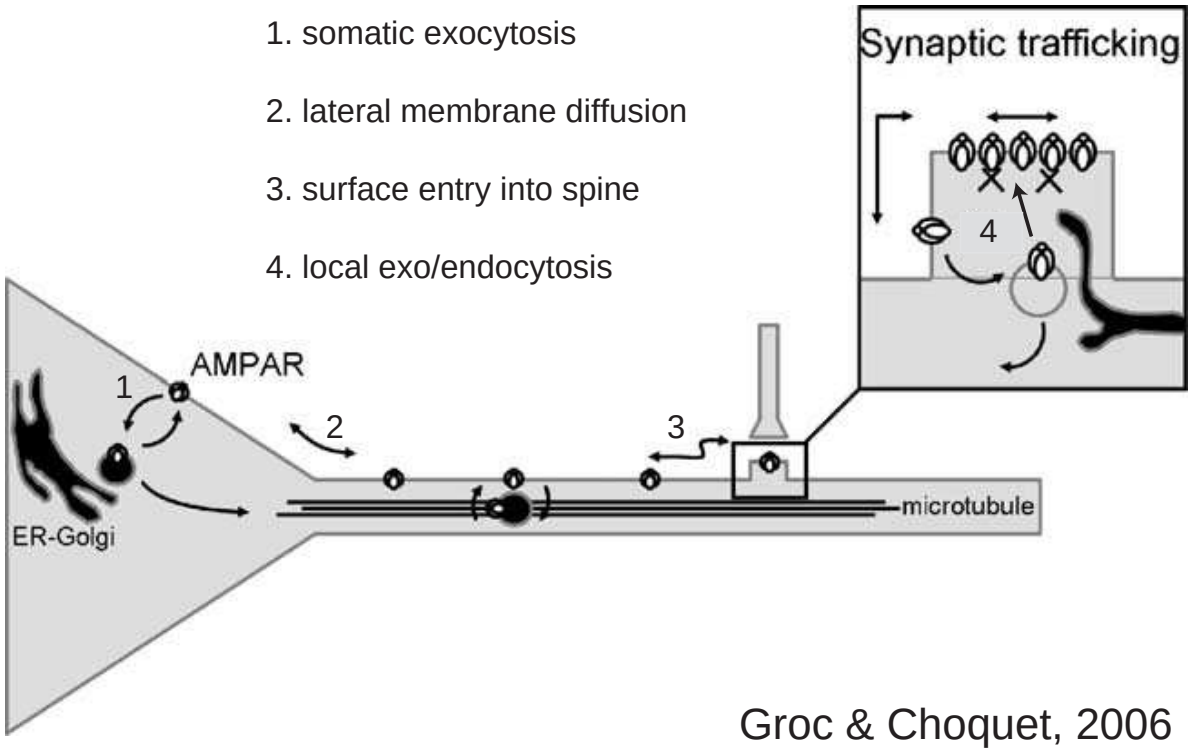
Passafaro et al., 2001



Adesnik et al., 2005



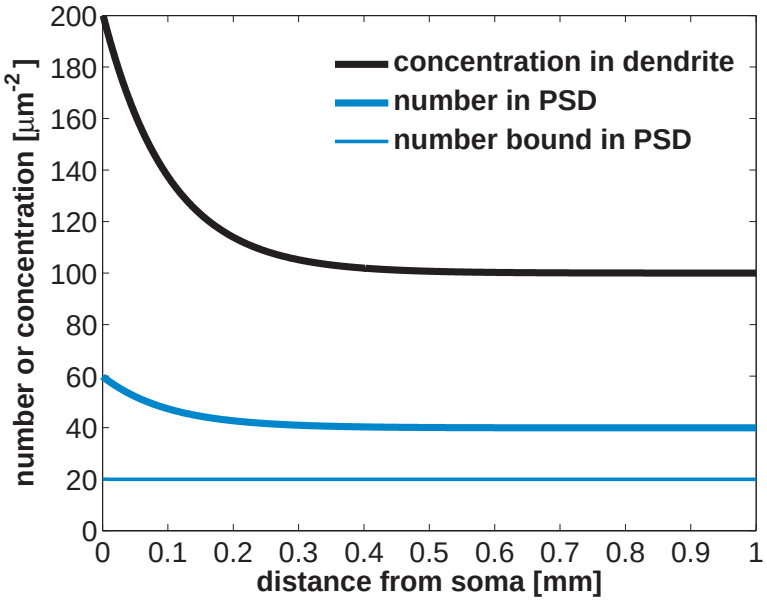
Long-range AMPA receptor trafficking

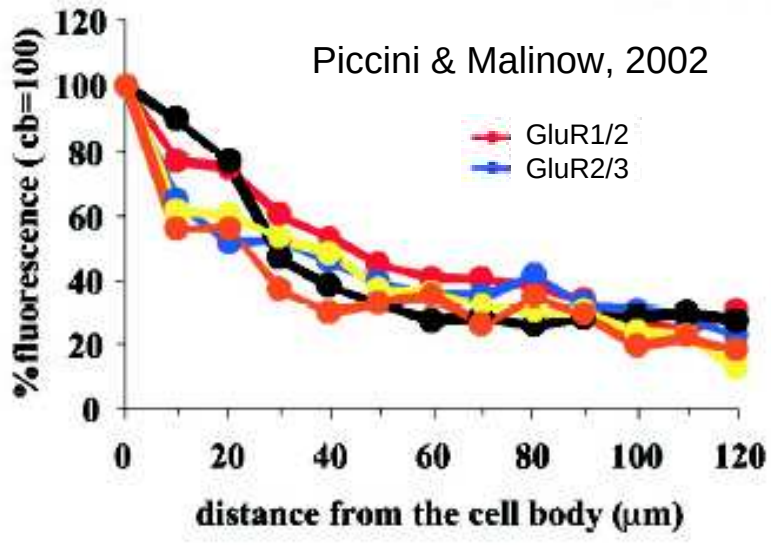


Groc & Choquet, 2006

- AMPARs trafficked in vesicles along microtubules?
- AMPARs diffuse from soma to synapse?

Steady-state AMPA receptor profiles for uniform cable

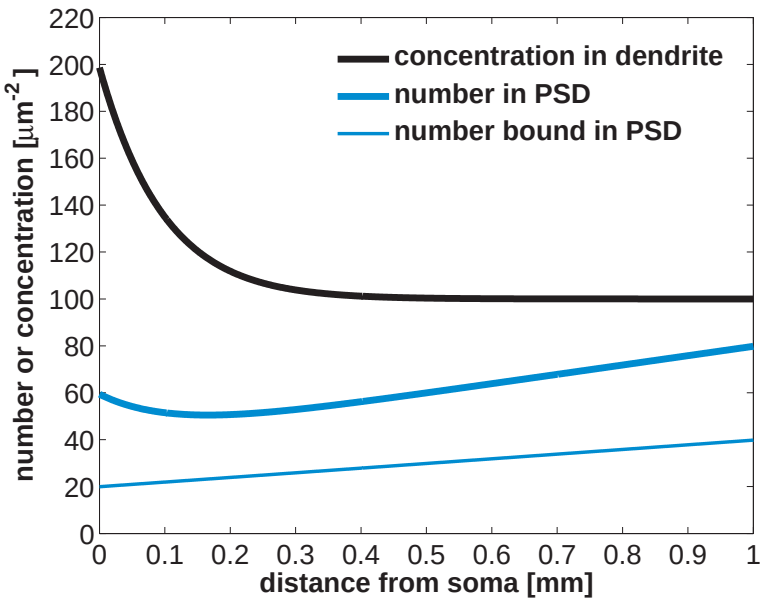




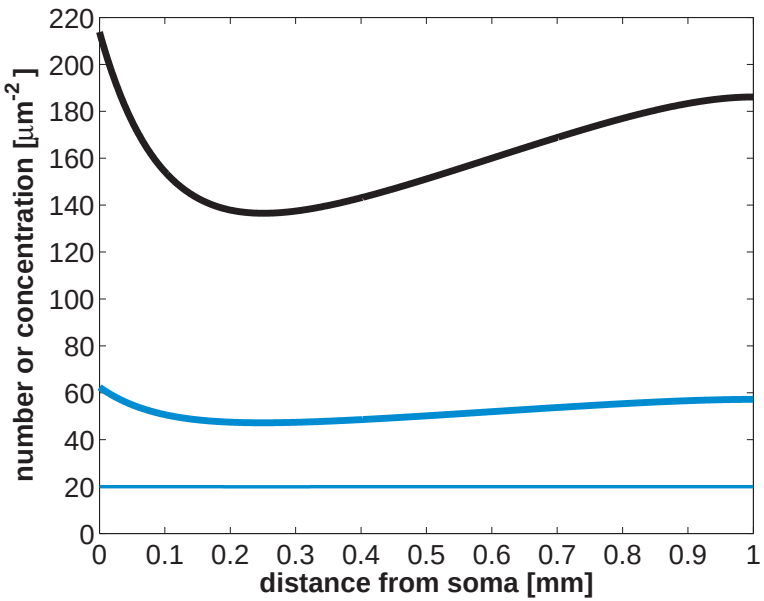
- 1,000 identical spines distributed uniformly along 1 mm dendrite
- Two sources of AMPARs
  - at soma
  - local intracellular delivery
- diffusion coefficient $D = 0.1\ \mu m^2 s^{-1}$

Profiles for nonuniform cable: synaptic democracy

PSD surface area
or spine density
increases linearly

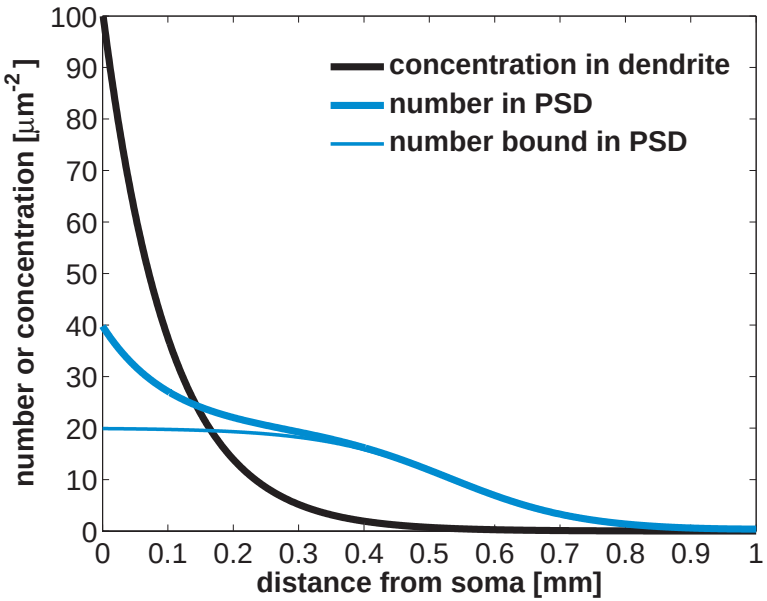


rate of delivery
or exocytosis
increases linearly

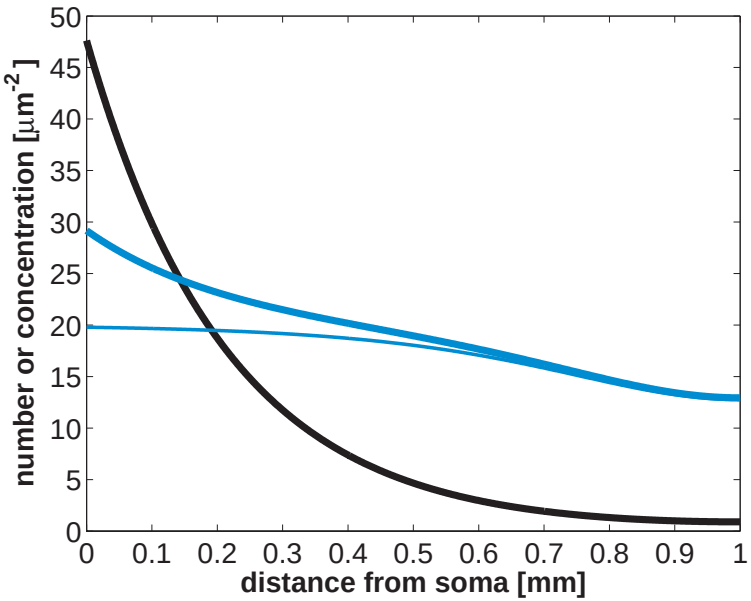


Profiles for uniform cable without intracellular delivery

$D = 0.1\ \mu m^2 s^{-1}$



$D = 0.45\ \mu m^2 s^{-1}$



- Mean time to reach distance X from soma $> \frac{X^2}{2D}$
- For $D = 0.45\ \mu m^2 s^{-1}$
  - $X = 100\ \mu m \Rightarrow \frac{X^2}{2D} \sim 3$ hr
  - $X = 1\ mm \Rightarrow \frac{X^2}{2D} \sim 300$ hr!

Intensive vs. extensive parameters

- Trafficking parameters categorized into two groups: whether or not localized changes in parameter produce nonlocal changes in steady-state synaptic AMPAR numbers

Intensive
(local effect only)

- PSD surface area a
- scaffolding concentration Z
- binding rate α
- unbinding rate β

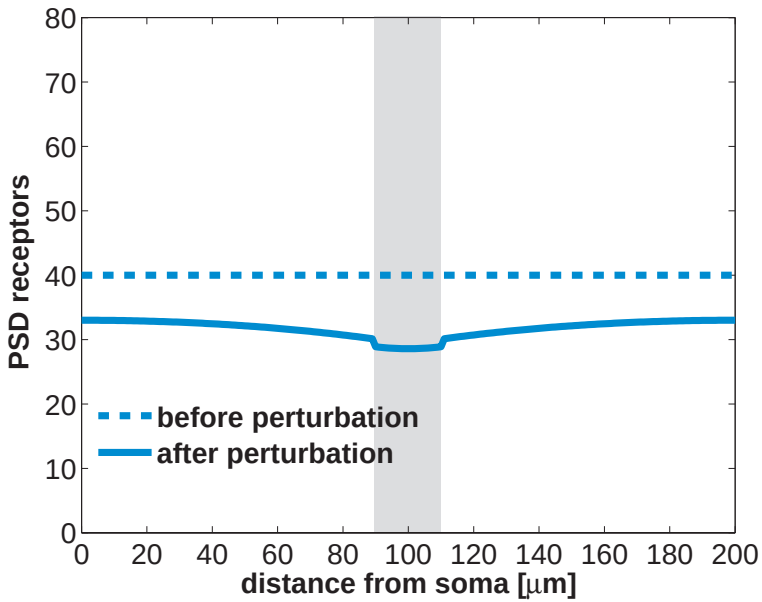
Extensive
(nonlocal effect)

- rate of exocytosis σ^{EXO}
- rate of endocytosis k
- intracellular delivery rate δ
- degradation rate σ^{DEG}

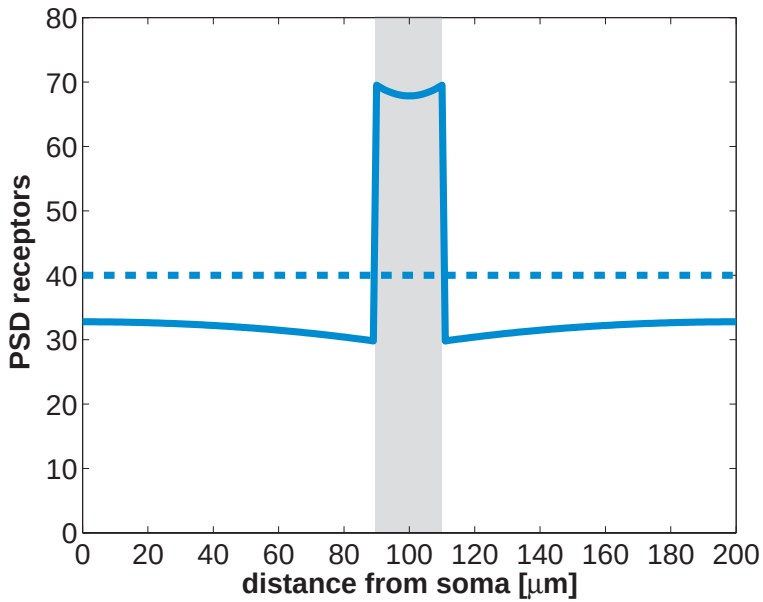
- Spine neck hopping rate Ω can be extensive, but not in current parameter regime ($\sigma^{\mathrm{EXO}} \gg \sigma^{\mathrm{DEG}}$)

Heterosynaptic dependence on constitutive recycling

10-fold reduction in rate of exocytosis in gray region



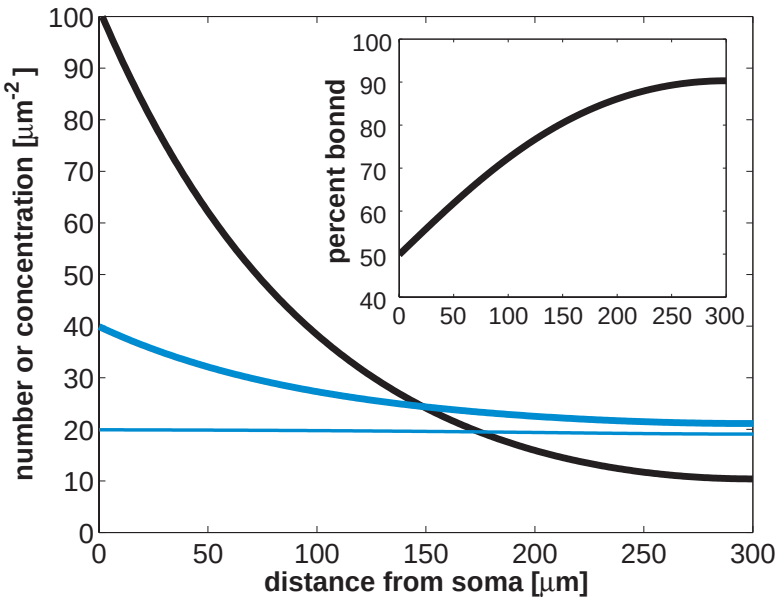
10-fold increase in rate of endocytosis in gray region



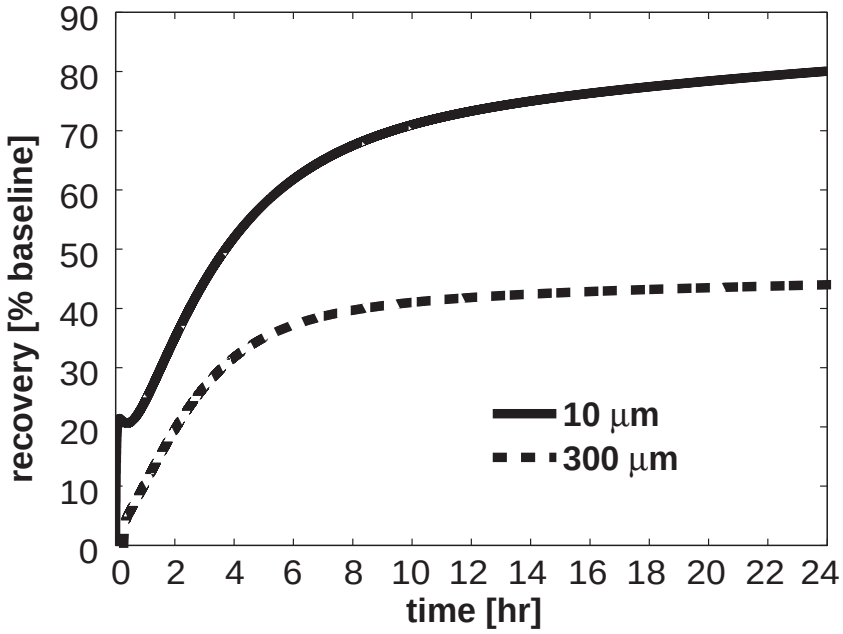
Simulation of photoinactivation

Assume

- no intracellular delivery but source at soma
- in steady-state $t < 0$
- at $t = 0$ all surface AMPARs instantaneously "inactivated"



Rate of recycling depends on distance from soma



- Fast exo/endocytosis is consistent with slow constitutive recycling
- Rate-limiting step is unbinding from scaffolding and diffusion from PSD

Summary

- Source of AMPARs at soma implies
  - exponential decay for uniform spines
  - synaptic democracy for nonuniform spines
- Need fast lateral diffusion to deliver AMPARs to distal synapses from soma
  - Takes too long?
- Local changes in recycling produce nonlocal changes in synaptic AMPAR numbers
  - Extensive vs. intensive trafficking parameters
- Constitutive recycling rate is distance-dependent when soma is only source of AMPARs
  - fast recycling at proximal synapses
  - slow recycling at distal synapses

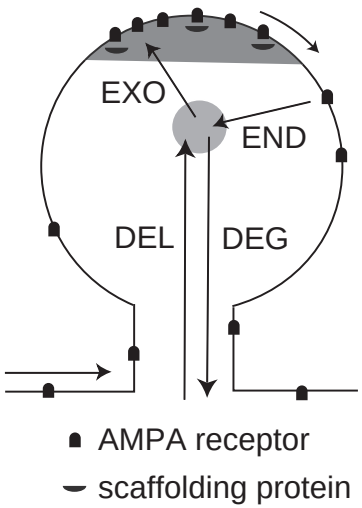
Model of trafficking at a single spine

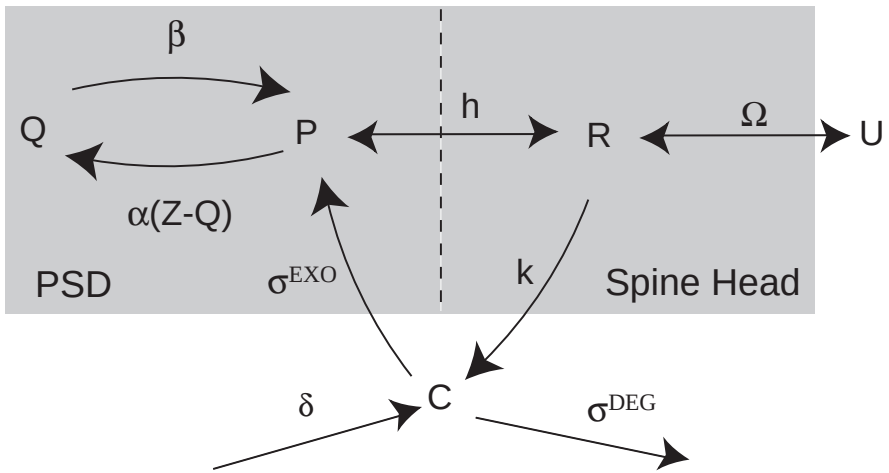
Spine head: $$\frac{dR}{dt} = \frac{1}{A}\left(\Omega[U-R] - kR - h[R-P]\right)$$

PSD unbound: $$\frac{dP}{dt} = \frac{h}{a}[R-P] - \alpha[Z-Q]P + \beta Q + \frac{\sigma^{\mathrm{EXO}} C}{a}$$

PSD bound: $$\frac{dQ}{dt} = \alpha[Z-Q]P - \beta Q$$

Intracellular: $$\frac{dC}{dt} = -\sigma^{\mathrm{EXO}} C - \sigma^{\mathrm{DEG}} C + kR + \delta,$$





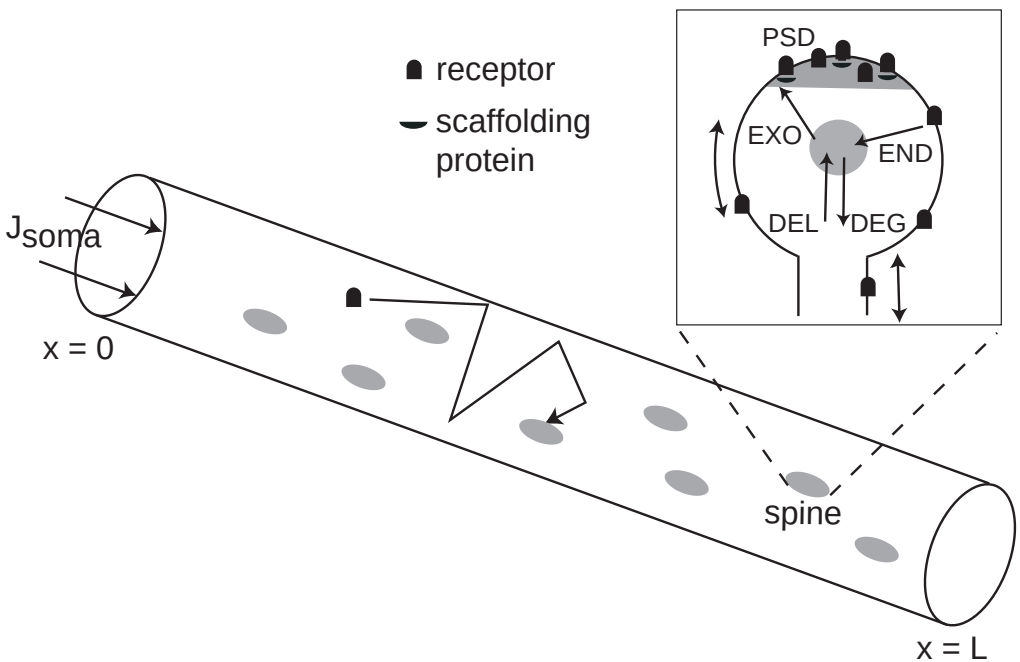
Model of trafficking along a spiny dendrite

$$\frac{\partial U}{\partial t} = D\frac{\partial^2 U}{\partial x^2} - \rho(x)\Omega(x)[U(x,t) - R(x,t)]$$

$$D\left.\frac{\partial U}{\partial x}\right|_{x=0} = -J_{\text{soma}}, \quad D\left.\frac{\partial U}{\partial x}\right|_{x=L} = 0.$$

D = diffusion coefficient, $\rho(x)$ = spine density at x

J_{soma} = surface flux from soma



Baseline parameter values

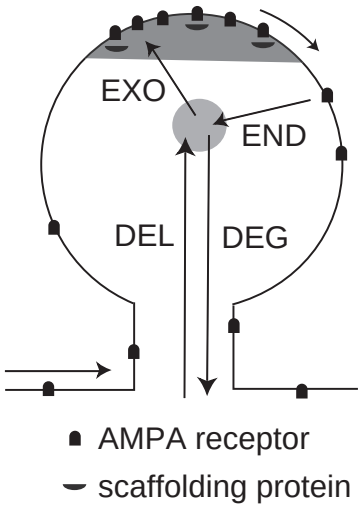
Parameter	Symbol	Value	Reference
Length of dendrite	L	1 mm	Sorra & Harris 2000
Circumference of dendrite	l	1 μm	Sorra & Harris 2000
Diffusion coefficient	D	0.1 $\mu m^2 s^{-1}$	Tardin et al. 2003
Spine density	ρ	1 μm^{-2}	Sorra & Harris 2000
Surface area of head	A	1 μm^2	Sorra & Harris 2000
Surface area of PSD	a	0.1 μm^2	Sorra & Harris 2000
Scaffolding concentration	Z	200 μm^{-2}	BE & Bressloff 2006
Binding rate	α	10^{-4} $\mu m^2 s^{-1}$	BE & Bressloff 2006
Unbinding rate	β	10^{-4} s^{-1}	BE & Bressloff 2006
PSD hopping rate	h	10^{-3} $\mu m^2 s^{-1}$	BE & Bressloff 2006
Spine neck hopping rate	Ω	10^{-3} $\mu m^2 s^{-1}$	BE & Bressloff 2006
Rate of endocytosis	k	10^{-3} $\mu m^2 s^{-1}$	Ehlers 2000
Rate of exocytosis	σ^{EXO}	10^{-3} s^{-1}	Passafaro et al. 2001
Degradation rate	σ^{DEG}	10^{-5} s^{-1}	O'Brien et al. 1999

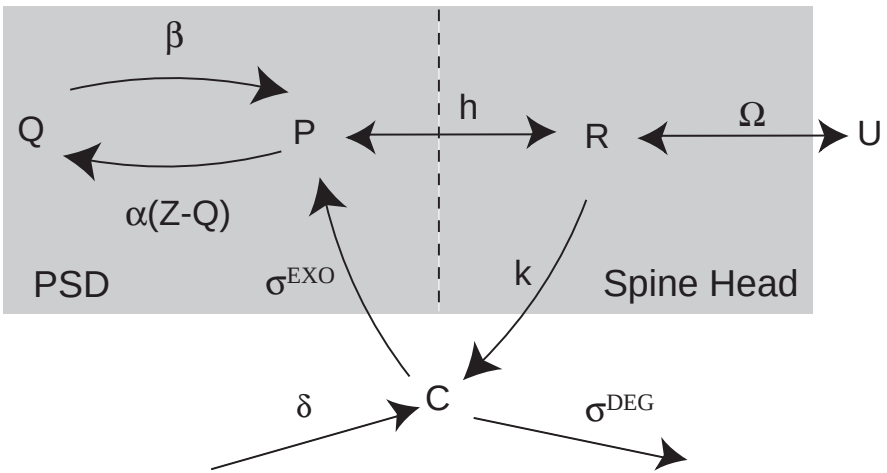
Steady-state at single spine

$$\sigma^{\mathrm{EXO}} C = \lambda[kR + \delta], \quad \lambda = \frac{\sigma^{\mathrm{EXO}}}{\sigma^{\mathrm{EXO}} + \sigma^{\mathrm{DEG}}}$$

$$P = \left[1 + \frac{\lambda k}{h}\right] R + \frac{\lambda \delta}{h}, \quad Q = \frac{\alpha P Z}{\beta + \alpha P}$$

$$R = \frac{\Omega U + \lambda \delta}{\Omega + k(1 - \lambda)}.$$





Steady-state dendritic concentration

$$D\frac{d^2U}{dx^2} - \rho\widehat{\Omega}U = -\rho\widehat{\Omega}r$$

$$\widehat{\Omega} = \frac{\Omega k(1-\lambda)}{\Omega + k(1-\lambda)}, \quad r = \frac{\sigma^{\mathrm{EXO}}\delta}{\sigma^{\mathrm{DEG}}k}$$

One can view

- $\widehat{\Omega}$ as effective spine neck hopping rate
- r as effective ESM receptor concentration

Solution for uniform cable: "cable" equation

- Assume all parameters are x-independent, then get "cable" equation for receptor trafficking

$$\frac{d^2U}{dx^2} - \Lambda_0^2 U(x) = -\Lambda_0^2 r, \quad \Lambda_0 = \sqrt{\frac{\rho\widehat{\Omega}}{D}}$$

- Solve using Green's function methods like standard cable equation for electrical current flow in passive dendrites

$$U(x) = \frac{J_{\text{soma}}}{D} \frac{\cosh(\Lambda_0[x-L])}{\Lambda_0 \sinh(\Lambda_0 L)} + r$$