

**MTH 370, Fall 2009**  
**Solutions to Homework 1**

1. (a)  $c_{n+1} = (1 - p)kc_n$   
 (b)  $c_n = ((1 - p)k)^n c_0$   
 (c)  $c_n = c_0 \Rightarrow p = 1 - 1/k$
  
2. (a)  $c_{n+1} = kc_n - h$   
 (b)  $c_n = k^n c_0 - h \sum_{i=0}^{n-1} k^i = k^n c_0 - h \frac{k^n - 1}{k - 1}$   
 (c)  $c_n = c_0 \Rightarrow k^n c_0 - h \frac{k^n - 1}{k - 1} = c_0 \Rightarrow h = (k - 1)c_0$
  
3. (a)  $c = 0$ , and  $c = \ln(r)$   
 (b)  $f(c) = rce^{-c} \Rightarrow f'(c) = (1 - c)re^{-c}$ , so  
 $c = 0$ :  $|f'(0)| = |r| = r$ , so stable when  $r < 1$ , unstable when  $r > 1$   
 $c = \ln(r)$ :  $|f'(\ln(r))| = |1 - \ln(r)|$ , so stable when  $1 < r < e^2$ , unstable when  $r < 1$  or  $r > e^2$   
 (c) Cobweb diagram:

