**MTH 370, Fall 2009**
**Midterm**

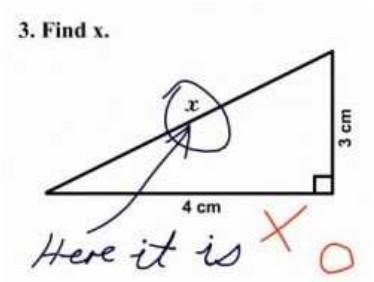
**Name:**

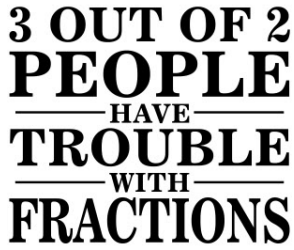
**Instructions:**

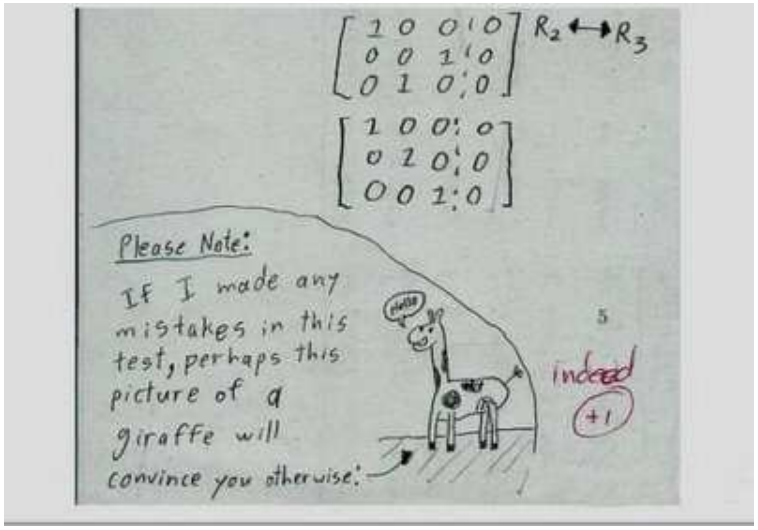
1. Print your name in the space provided above.
2. There are two problems. Do these problems by hand (no calculators, computers, etc.) and show your work on the pages provided (no additional scratch paper).
3. You may begin the exam when Berton indicates it is 12:40pm.
4. All exams must be returned by the end of the regular class period (1:30 pm).
5. Try to enjoy yourself.

Here are some useful reminders about taking derivatives:

| Rule | Formula | Example |
|---|---|---|
| Product | $(f(x)g(x))' = f'(x)g(x) + f(x)g'(x)$ | $(x\ln(x))' = x'\ln(x) + x\ln'(x) = \ln(x) + 1$ |
| Quotient | $\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$ | $\left(\frac{x}{x+1}\right)' = \frac{x'(x+1) - x(x+1)'}{(x+1)^2} = \frac{1}{(x+1)^2}$ |
| Chain | $(f(g(x)))' = f'(g(x))g'(x)$ | $(e^{-x^2})' = e^{-x^2}(-x^2)' = -2xe^{x^2}$ |



1. Consider the Beverton-Holt model,

$$x_{n+1} = \frac{rx_n}{x_n + 1} \quad (r > 1), \tag{1}$$

which has been used to successfully model some fish populations.

(a) (20%) Find the two fixed points of (1).

(b) (20%) Determine the stability of the fixed points.

Scratch paper.

2. Consider the following age-structured population model,

$$\mathbf{x}_{n+1} = M\mathbf{x}_n \quad \left( M = \begin{bmatrix} 1 & 0.5 \\ 1 & 0.5 \end{bmatrix}, \quad \mathbf{x}_n = \begin{bmatrix} A_n \\ J_n \end{bmatrix} \right). \tag{2}$$

(a) (20%) Compute the trace, determinant and inverse of $M$.

(b) (30%) Compute the eigenvalues and eigenvectors of $M$.

(c) (10%) Will this population eventually go extinct?

Scratch paper.